Forecasting the Market for Cellular Mobile Services by Linear Regression Models

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We consider the problems of explaining and forecasting the penetration and the traffic in cellular mobile networks. To this end, we create two regression models, viz. one to predict the penetration from service charges and network effects and another one to predict the traffic from service charges and diffusion and adoption effects. The results of the models can also be combined to compute the likely evolutions of essential characteristics such as Minutes of Use (MoU), Average Revenue per User (ARPU) and total revenue. Applying the models to 28 markets throughout the world we show that they perform very well. Noting the significant qualitative differences between these markets, we conclude that the model has some universality in that the results are comparable for all of them.

1 Introduction

This paper deals with econometric methods for forecasting the number of subscribers and the voice traffic in cellular mobile networks.

Forecasting is an essential part of planning and maintaining telecommunications networks. For this reason forecasting has been a concern mainly for network operators. Today, however, forecasting is a concern for a wider community including equipment manufacturers, service providers and government regulators. Equipment manufacturers need to support their customers and plan their development projects; service providers need to plan and maintain their systems to ensure adequate quality and stay competitive; and regulators need to make independent estimates of the demand to secure the availability of essential services and to adequately judge complaints from network operators, service providers or end users. At the same time, forecasting is becoming increasingly complex as deregulation and competition not only encourages faster development and deployment of new services and technologies but also tend to reduce their life times.

Our primary aim is to consider a market for cellular mobile services and *forecast* penetration and traffic with a *long-term* perspective. As a secondary aim, we also wish to *explain* the evolution of these quantities over time and to *predict* other, related quantities of interest. In this context, a 'market' is a country or an operator and 'long-term' refers to a couple of years.

In principle, we are interested in all kinds of services, such as voice, video, messaging and data; hence our models do not include service specific characteristics. As for numerical examples, however, voice (and possibly SMS) is (are) the only service(s) for which sufficient statistics are available.

In this context it is worth recalling that voice still is the most important service in just about all cellular networks. To see this, consider the possibly most advanced markets in the world, viz. South Korea (represented by SK Telecom [1]) and Japan (represented by NTT DoCoMo [2]).[1] In terms of *revenues* [1] reports a ratio of three to one between voice ARPU and data ARPU (KRW 29,516 to KRW 10,689). We also note that about 40 % of the data ARPU is related to SMS. The same figure in [2] is about four to one (JPY 5,030 to JPY 1,880). In terms of *volumes* [1] reports a per-user monthly voice volume of 20 Megabytes (MoU 197 and 13.4 kbps). The same figure in [2] is 15 Megabytes (MoU 149 and 13.4 kbps). We also note that these numbers double if inbound calls are included and double again if IP overhead is added. We are not aware of any official figures for data volumes, but it is clear that the volumes from services like SMS are extremely small. Finally, in terms of *growth* [1] reports voice volumes up by 5.6 % (MoU up from 194 to 197 and users up from 18.789 million to 19.530 million). The same figure in [2] is 3.4 % (MoU down from 151 to 149 and users up from 48.825 million to 51.144 million). We are not aware of any official figures for data growth rates.

Considering these numbers and the fact that other markets are far behind, we may safely conclude that voice is important; both in terms of revenues and volumes, and that voice is growing. Having said that, we firmly believe that other services will dominate in the future and we are looking forward to obtaining data which allows us to apply our models to these services.

The remainder of the paper is organised as follows: We begin in Section 2 with brief overviews of established forecasting methods and of the relevant litera-

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[1] *These operators were chosen with no other intention but to provide real numbers from the* leading *operators on the* leading *markets.*

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ture. In Section 3 we formulate our problem and present our initial model. The parameters are discussed further in Section 4 and Section 5 after which the final models are obtained in Section 6. The results are applied to real data in Section 7 where we explain and forecast penetration, traffic and other quantities of interest. Finally we sum up our results in Section 8.

2 Preliminaries

2.1 Methods

The number of forecasting methods is possibly as large as (or larger than) the number of persons involved in forecasting. Using the taxonomy of financial markets, there are *analytical* methods and *technical* methods. In simple terms, analytical models (like regression models) are based on mathematical models of the *relationship between observed and explanatory data* whereas technical methods (like curve fitting, ARIMA or Kalman) are based on mathematical models of the *observed data itself*. See, eg. [3–5] for a general introduction. In addition there are models for *proposed* future services. For more details see, eg. [6,7].

We consider existing services with a known history. Analytical models of these services add an understanding of the factors behind the observations and this tends to make them more robust. Technical models, on the other hand, are simple to apply and some of them do not require long series of historic observations. The two methods thus both have their *pros* and *cons*, hence they complement each other rather than compete with each other.

2.2 Literature

Because of our long-term perspective and our ambition to explain the evolution, we settle for an analytical method. Like most such methods we rely on product form models solved by ordinary least squares linear regression. The literature in this area is surprisingly sparse.

The problem of explaining the evolution of the penetration can be referred to as a diffusion problem. The classical work in this area is the Bass model [8] where a saturation level and two diffusion parameters called innovation and imitation are estimated from observed data. A thorough review of diffusion models and their applications, with particular emphasis on telecommunications, is found in [9]. In addition, the Bass model was used by [10] but with an externally estimated saturation level. We have tried the pure model with unsatisfactory results.

As for economic approaches, a thorough survey of economic studies modelling cross-country mobile telephone diffusion is found in [11]. This paper also examines the significance of 18 different socio-economic factors in 180 different countries. The study concludes that the most significant factor is the network effect. An explanatory model with almost the same variables as in our model is proposed by [12]. The main differences are that [12] treats income and cost as two variables whereas we use cost relative to income as one variable and that the per-minute costs in [12] are based on official model calls whereas we use actual de facto expenses.

The problem of explaining the evolution of the traffic by economic models is even less well represented in the literature. The works in [4,5] consider international, fixed telephony. These specific models, which include variables like 'number of automatic exchanges' and 'bilateral trade', are too old and too specific for our purposes. The only other work we have come across is our own [13,14]. The present paper is an improved, updated and extended version of the latter.

We conclude the literature survey by mentioning the two special issues of *Telektronikk* devoted to forecasting telecommunications, one from 1994 [15] and another one from 2004 [16].

2.3 Contribution

The major contribution of the present work is that (i) we consider *both* penetration and traffic, (ii) we consider a *combination* of (relative) service charges, network effects, and diffusion and adoption effects, and that (iii) our models are applied to *several* markets, (iv) our models yield very *accurate* results, yet (v) our models only need *one* external variable.

3 Basic Model

Let X denote the quantity of interest, ie. penetration or traffic. Based on our earlier work [13,14] and inspired by the examples in [4, pp.11–13] and [5, pp. 19–20], we selected a basic model of X with both economic and non-economic variables, viz. the actual cost C of the service and the penetration P on the market

$$X = g(C) \cdot h(P) \tag{1}$$

where g and h are functions and X, C and P are indexed by time (eg. year). With X representing penetration, equation (1) thus means that penetration is a function of cost at this and/or previous years and penetration at previous years. Similarly, with X representing traffic, equation (1) means that traffic is a function of cost and penetration at this and/or previous years.

It is noted that, although it may be desirable to include more variables, we are limited by the fact that the number of parameters must be significantly lower than the number of historic observations available in order for a model to be meaningful. We are also limited by the kind of data that is publicly available.

4 Modelling Cost

The decision to subscribe to and to use mobile services depends on the cost for using the mobile services, including subscription fees, usage tariffs and the cost of the terminal. We can assume that mobile services are like most other goods in that they have a downward sloping demand curve, implying that penetration and traffic increase when prices go down.

The question is how to measure the cost for mobile services. In most markets, there are several operators each of which offers a large number of different subscription fees and tariff structures. Moreover, most deals with customers in the business segment are not even disclosed to the public. These circumstances make it very difficult in practise to measure average costs on mobile markets.

However, we have used the fact that the cost for the users must be equal to the operators' revenues – they are just two sides of the same coin. Revenues are often measured as (monthly) ARPU and usage is often measured as (monthly) MoU. A simple and approximate measure of the actual cost per minute may thus be obtained as ARPU divided by MoU.

Since ARPU often includes both subscription fees and usage tariffs (and sometimes also terminal costs), our measure thus includes the combined effect from all such charges.

Cost is, however, also a relative measure in the sense that people have limited financial resources and need to make decisions on how much of the available resources to spend on mobile services. Spending on mobile services thus depends not only on cost as such, but also on the size of the consumption budget and on individual preferences between consumption alternatives.

The consumption budget may change over time and the preferences may differ depending on the size of this budget. We have used the gross domestic product per capita (GDP) as the measure of the overall spending power in the market. GDP has advantages over many other measurements like, eg. consumer spending index, as it is commonly defined and publicly available in most countries. An approximate measure of the actual cost per minute relative to the spending budget may thus be obtained as (ARPU / MoU) / GDP.

The suggested metric thus 'aggregates' price and GDP into one variable whereas they typically are treated as two separate variables. The advantage with this approach is that one variable less means one more degree of freedom, and this will increase the precision of the computed parameters. In addition, the suggested metric will by itself be adjusted for inflation and it is independent of the currency used. The approach may also be viewed as a compromise between the standard approach of including the GDP and the finding in [11] that GDP is relatively insignificant. Similar conflicts are also mentioned in [12].

A possibly more important aspect is, however, the fact that we consider *actual costs* as reported by operators whereas other works tend to rely on *official statistics* such as (some) peak-time charge per minute (eg. [11]) or (some) subscription fee (eg. [12]). This is a significant difference because our metric will account for a whole range of important factors such as form of payment (prepaid, eg. typically implies additional per-minute charges but no subscription fees), charging scheme (different forms of 'flat rate' are, eg. available on many markets) and user segment (different discounts may be offered to, eg. large corporations).

The relationship between cost C and demand D can be modelled in different ways. The classical model with a constant elasticity reads

$$D = K \cdot C^{a} \tag{2}$$

where K is a scaling constant and a corresponds to the elasticity of demand with respect to cost. With $a < 0$ the model may be interpreted such that the demand will increase by a % if the cost decreases by 1 % and vice versa. The two unknown parameters K and a can be estimated from historic observations of D and C.

The model in Eq. (2) is a special case of willingness to pay (WTP) models. (To avoid possible confusion we point out that the *concept* of WTP is used in *hypothetical* situations whereas we will use *models* of WTP to characterise *actual* outcomes.) This class of models also includes models based on distributions

$$D = K \cdot \overline{F}(C) \tag{3}$$

where K again is a scaling constant and $\overline{F}$ is the survivor function of a probability distribution. Typical WTP distributions include eg. the log normal distribution and the negative exponential distribution,

$$D = K \cdot e^{aC} \tag{4}$$

where $-1/a$ is the average price a user is willing to pay.

We adopt the simpler model Eq. (2) for traffic because it is a continuous entity where a marginal increase may have a marginal value which is hard to model by distributions. On the other hand, we adopt the more advanced model Eq. (4) for penetration because a subscription is a discrete entity and the result of elaborated decisions based on cost, budget and personal preferences.

5 Modelling Penetration

The decision to subscribe to and to use mobile services also depends on the penetration. Like most works in this area, we can assume that there is a network effect (which means that the higher the penetration, the more attractive is the network) and diffusion and adoption effects (which mean that the keenest users will be the first to join and vice versa).

A simple approach to characterise the network effect is attributed to Metcalfe [17] who suggested that the value of a network is proportional to the square of the number of users connected to it. This statement can be generalised to a model of constant elasticities similar to the one in Eq. (2),

$$D = K \cdot P^{b} \tag{5}$$

where, as before, K is a scaling constant and b is the elasticity of demand with respect to penetration. The model may be interpreted such that the demand will increase by b % if the penetration increases by 1 % and vice versa. The two unknown parameters K and b can again be estimated from historic observations of D and P.

A similar approach may be used to model diffusion and adoption effects. A more advanced version, however, is to apply such a model to a weighted penetration where the penetration is recomputed to reflect not just 'total users' but rather 'total user interest'. To this end, we suggest to characterise the interest of a user by the cost of the service at the time the user joins. Such a model may then be written as

$$D = K \cdot (\Sigma_y \, C_y \, \Delta P_y)^{b} = K \cdot W^{b} \tag{6}$$

where K again is a scaling constant, C_y is the cost year y, ΔP_y is the change in penetration during year y and b as before is the elasticity of demand with respect to weighted penetration.

Finally, it may be preferred to explicitly account for saturation effects related to eg. the number of inhabitants I in a country. Our formal model reads

$$D \propto e^{cS} \tag{7}$$

where c is a constant and the saturation S is defined as the penetration P less the population I, $S_y = P_{y-1} - I_y$. Note that the saturation can be negative, zero or positive; with $c < 0$, a negative saturation (less than one SIM card per inhabitant) will encourage growth (Eq. (7) > 1) and a positive saturation (more than one SIM card per inhabitant) will discourage growth (Eq. (7) < 1).

We prefer the simpler model Eq. (5) for penetration because increasing penetration is not only a result of increasing value (the network effect) but also related to increasing awareness (the imitation effect). In this case we try with and without the saturation model Eq. (7). On the other hand, we prefer the more advanced model Eq. (6) for traffic because it contains more information and we can assume that less keen subscribers always will be less active and vice versa.

6 Final Models

The suggested models of penetration P (average penetration over a year) and traffic T (average number of traffic minutes per month) are thus obtained by inserting Eq. (4) and (5) (possibly complemented by Eq. (7)) and Eq. (2) and (6) respectively into Eq. (1). We call these models the 'A'-models,

$$P_y^{A} = K \cdot e^{aC_y} \cdot P_{y-1}^{b} \tag{8}$$

$$P_y^{A'} = K \cdot e^{aC_y} \cdot P_{y-1}^{b} \cdot e^{cS_y} \tag{9}$$

$$T_y^{A} = K \cdot C_y^{a} \cdot W_y^{b} \tag{10}$$

where subscript y refers to year.

It is noted that Eq. (8) or Eq. (9) can be used recursively to forecast the penetration P_{y+1} for different assumptions regarding costs C_{y+1} (and, in the latter case, inhabitants I_{y+1}) after which Eq. (10) can be used to forecast the traffic T_{y+1} for the same assumptions regarding costs C_{y+1}.

The results of these forecasts can be used to calculate the corresponding evolutions of MoU U_{y+1}, adjusted voice ARPU A'_{y+1} and adjusted total voice revenues R'_{y+1} as follows

$$U_y = T_y \, / \, P_y \tag{11}$$

$$A'_y = C_y \cdot U_y \tag{12}$$

$$R'_y = P_y \cdot A'_y \tag{13}$$

Note that voice ARPU A' and total voice revenues R' are not given in nominal currency but adjusted with respect to inflation and GDP per capita.

The unknown parameters K, a, b and c can be solved for by rewriting Eq. (8) – Eq. (10) as multiple linear regression models

$$\ln\left(P_y^A\right) = \hat{K} + a(C_y) + b\ln(P_{y-1}) \tag{14}$$

$$\ln\left(P_y^{A'}\right) = \hat{K} + a(C_y) + b\ln(P_{y-1}) + c(S_y) \tag{15}$$

$$\ln\left(T_y^A\right) = \hat{K} + a\ln(C_y) + b\ln\left(\sum_y C_y \Delta P_y\right) \tag{16}$$

from which the unknowns can be computed by the ordinary least squares technique.

We require that $K > 0$, $a < 0$, $b > 0$ and $c < 0$. The first requirement is obvious and the other requirements are motivated by the fact that interest is discouraged by higher costs or increasing saturation but encouraged by increasing penetration.

If required, we repeatedly reject the oldest remaining observation until these requirements are met. The reason for this is that 'invalid' models most likely are attributed to 'invalid' data. There are many reasons for why data can be invalid.

- A simple reason is eg. that operators tend to change the way parameters like ARPU and penetration are reported. Examples of such changes include reporting of late payment fees and policies toward passive subscribers. Sometimes these changes may pass unnoticed or be impossible to correct.

- Some more complex reasons are eg. that operators may have expanded the coverage or governments may have changed the regulations. Such changes may be very significant but lie outside our model.

- Finally, customers may change behaviour when prices and penetration change by several orders of magnitude. (Note, however, that this does not happen in case of eg. electricity.) A possible example of such a change is 'sudden' substitution effects from fixed telephony once costs become comparable.

The idea behind rejecting old observations is thus to remove observations regarding old reporting, old coverage, old regulations etc until the model makes sense. This will not guarantee but at least improve the chances that all 'external' conditions are stable in the remaining time period.

Another approach to handle more or less 'difficult' cases is to use different models. The suggested, alternative models of penetration P and traffic T are obtained by sticking to the simple cost model, Eq. (2), and the simple penetration model, Eq. (5). We call the resulting models the 'B'-models,

$$P_y^{\mathrm{B}} = K \cdot C_y^a \cdot P_{y-1}^b \tag{17}$$

$$P_y^{\mathrm{B}'} = K \cdot C_y^a \cdot P_{y-1}^b \cdot e^{cS_y} \tag{18}$$

$$T_y^{\mathrm{B}} = K \cdot C_y^a \cdot P_y^b \tag{19}$$

where subscript y again refers to year. Similar to above, the unknown parameters K, a, b and c can be solved for by rewriting the models as multiple linear regression models

$$\ln\left(P_y^{\mathrm{B}}\right) = \hat{K} + a\ln(C_y) + b\ln(P_{y-1}) \tag{20}$$

$$\ln\left(P_y^{\mathrm{B}'}\right) = \hat{K} + a\ln(C_y) + b\ln(P_{y-1}) + c(S_y) \tag{21}$$

$$\ln\left(T_y^{\mathrm{B}}\right) = \hat{K} + a\ln(C_y) + b\ln(P_y) \tag{22}$$

from which the unknowns again can be computed by the ordinary least squares technique.

7 Examples

We will now apply the models to $M = 28$ different markets. It is emphasised that the markets and the numbers are provided by the author only as examples and that they do not, in any way, represent official business outlooks.

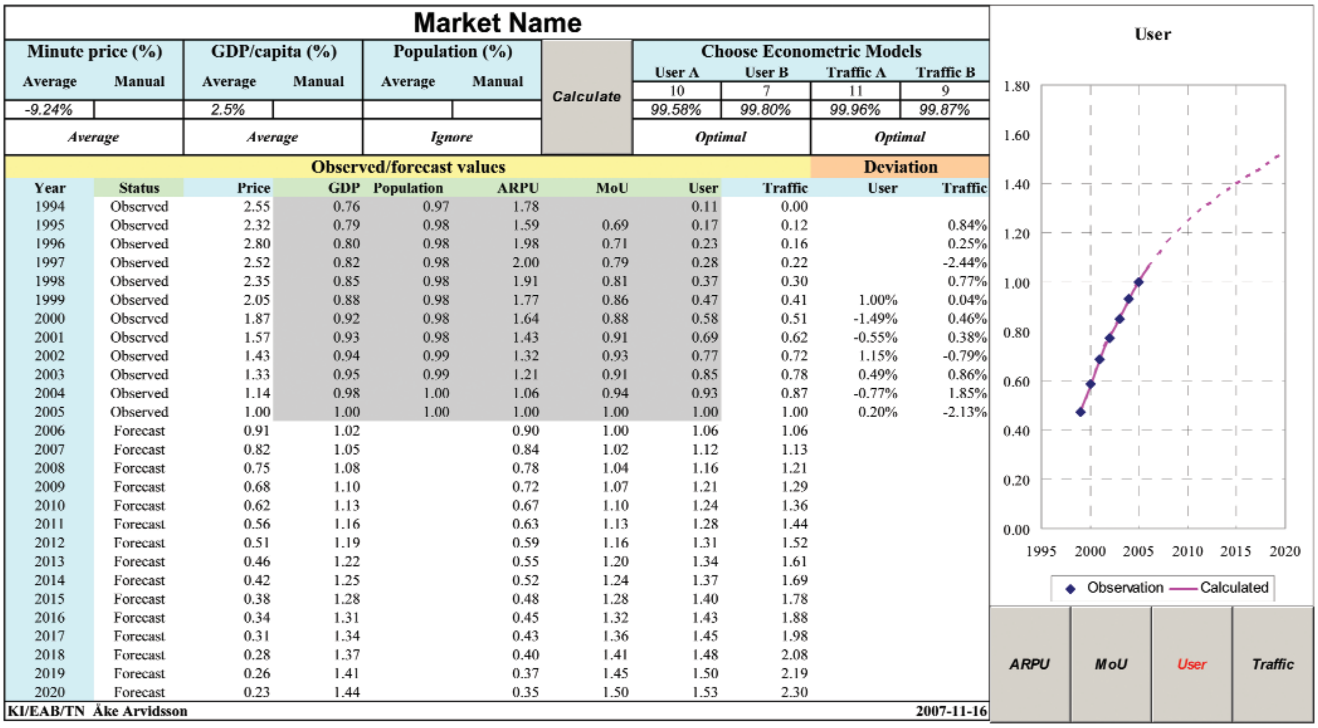
The starting point is tables of penetration $P_{m,y}$, ARPU $A_{m,y}$, MoU $U_{m,y}$, GDP $G_{m,y}$ and inhabitants $I_{m,y}$ for all markets $m = 1, ..., 28$ and over eleven years $y = 1995, ..., 2005$ (except for some markets where the tables span shorter times). We also remark that the data series for Norway are shorter and obtained from different sources.

This data is used to compute additional tables of cost

$$C_{m,y} = A_{m,y} \,/\, U_{m,y} \,/\, G_{m,y}, \tag{23}$$

weighted penetration

$$W_{m,y} = C_{m,1995}P_{m,1995} + \sum_{v=1996}^{y} C_{m,v}(P_{m,v} - P_{m,v-1}) \tag{24}$$



Market Name

Minute price (%)		**GDP/capita (%)**		**Population (%)**		**Calculate**	**Choose Econometric Models**			
Average	**Manual**	**Average**	**Manual**	**Average**	**Manual**		**User A**	**User B**	**Traffic A**	**Traffic B**
-9.24%		2.5%					10	7	11	9
							99.58%	99.80%	99.96%	99.87%
Average		*Average*		*Ignore*			*Optimal*		*Optimal*	

Observed/forecast values									**Deviation**	
Year	**Status**	**Price**	**GDP**	**Population**	**ARPU**	**MoU**	**User**	**Traffic**	**User**	**Traffic**
1994	Observed	2.55	0.76	0.97	1.78		0.11	0.00		
1995	Observed	2.32	0.79	0.98	1.59	0.69	0.17	0.12		0.84%
1996	Observed	2.80	0.80	0.98	1.98	0.71	0.23	0.16		0.25%
1997	Observed	2.52	0.82	0.98	2.00	0.79	0.28	0.22		-2.44%
1998	Observed	2.35	0.85	0.98	1.91	0.81	0.37	0.30		0.77%
1999	Observed	2.05	0.88	0.98	1.77	0.86	0.47	0.41	1.00%	0.04%
2000	Observed	1.87	0.92	0.98	1.64	0.88	0.58	0.51	-1.49%	0.46%
2001	Observed	1.57	0.93	0.98	1.43	0.91	0.69	0.62	-0.55%	0.38%
2002	Observed	1.43	0.94	0.99	1.32	0.93	0.77	0.72	1.15%	-0.79%
2003	Observed	1.33	0.95	0.99	1.21	0.91	0.85	0.78	0.49%	0.86%
2004	Observed	1.14	0.98	1.00	1.06	0.94	0.93	0.87	-0.77%	1.85%
2005	Observed	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.20%	-2.13%
2006	Forecast	0.91	1.02		0.90	1.00	1.06	1.06		
2007	Forecast	0.82	1.05		0.84	1.02	1.12	1.13		
2008	Forecast	0.75	1.08		0.78	1.04	1.16	1.21		
2009	Forecast	0.68	1.10		0.72	1.07	1.21	1.29		
2010	Forecast	0.62	1.13		0.67	1.10	1.24	1.36		
2011	Forecast	0.56	1.16		0.63	1.13	1.28	1.44		
2012	Forecast	0.51	1.19		0.59	1.16	1.31	1.52		
2013	Forecast	0.46	1.22		0.55	1.20	1.34	1.61		
2014	Forecast	0.42	1.25		0.52	1.24	1.37	1.69		
2015	Forecast	0.38	1.28		0.48	1.28	1.40	1.78		
2016	Forecast	0.34	1.31		0.45	1.32	1.43	1.88		
2017	Forecast	0.31	1.34		0.43	1.36	1.45	1.98		
2018	Forecast	0.28	1.37		0.40	1.41	1.48	2.08		
2019	Forecast	0.26	1.41		0.37	1.45	1.50	2.19		
2020	Forecast	0.23	1.44		0.35	1.50	1.53	2.30		

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Figure 1 Forecasting spreadsheet

Market	Model	R^2	$\hat{K}$	$-a$	t-rat.	b	t-rat.	$-c$	t-rat.	Val.	Rej.
Canada	A	1.00	1.83	1.04	5.51	0.45	4.90			9	0
United States	A	1.00	1.06	0.16	1.23	0.82	13.2			11	0
Argentina	A	0.99	2.35	2.58	4.35	0.42	4.00			10	0
Brazil	A	1.00	1.32	0.66	2.31	0.83	26.0			10	0
Colombia	A	0.97	1.03	0.74	1.18	0.79	5.94			10	0
Chile	A	1.00	1.64	2.08	14.1	0.52	17.9			10	0
Mexico	A′	1.00	1.31	1.74	7.14	0.67	10.8	0.01	1.19	10	0
China	B	1.00	1.70	0.07	2.97	0.69	35.3			11	6
India	B	1.00	0.08	0.72	2.91	0.50	3.03			10	5
Japan	B	1.00	1.28	0.07	0.54	0.76	30.3			10	3
Korea	A	0.99	3.36	1.27	0.71	0.17	0.29			8	0
Malaysia	A	0.99	1.36	1.01	2.18	0.66	4.98			10	0
Taiwan	A′	0.99	0.90	0.89	1.56	0.83	6.62	0.04	4.66	10	2
Thailand	A	0.94	2.25	1.59	1.30	0.61	2.12			8	0
France	A′	1.00	1.37	0.89	3.97	0.67	8.96	0.01	2.95	9	0
Germany	A′	1.00	0.96	1.50	3.36	0.91	7.08	0.01	4.30	9	0
Italy	A	1.00	3.35	1.83	4.16	0.41	4.68			9	0
Netherlands	A	1.00	2.25	1.84	7.58	0.39	7.18			9	0
Norway	A	0.98	5.21	0.22	1.41	0.67	9.66			8	0
Spain	B	1.00	0.82	0.24	7.46	0.49	35.9			9	3
Sweden	A′	1.00	0.59	0.55	0.74	0.84	2.60	0.04	2.96	10	0
United Kingdom	A′	1.00	1.33	1.16	5.02	0.79	5.49	0.01	1.62	9	0
Hungary	A′	1.00	0.55	0.79	1.25	0.87	6.47	0.06	2.69	10	0
Israel	A	0.99	0.85	0.40	1.71	0.79	22.1			9	0
Poland	B	1.00	0.09	0.26	2.23	0.74	19.7			10	3
Russia	A′	1.00	0.18	0.39	3.04	0.99	40.5	0.01	1.71	10	0
South Africa	A	1.00	1.44	0.78	1.09	0.77	8.75			8	0
Turkey	A′	1.00	0.60	0.71	3.62	0.87	9.02	0.01	1.46	8	1

Table 1 Results from fitting the penetration model

and traffic

$$T_{m,y} = M_{m,y} \, P_{m,y} \tag{25}$$

for these years. For the penetration model, the costs for each market are finally normalised by the highest cost on that market to enable comparisons between different markets.

The complete model was implemented in Visual Basic on an Excel spreadsheet. The sheet, depicted in Figure 1 consists of three parts, viz. mode selection (top), a plot area (right) and a data area (bottom).

- The mode selection part allows the user to set different scenarios for the evolution of price, GDP and population (average percentage, other percentage or external forecast) and choose between different models (A, A′, B or B′ for penetration and A or B for traffic).

- The plot area provides graphical representations of the evolution of ARPU, MoU, penetration or traffic as selected by the buttons.

- The data area contains the input and the output. The input (grey area) consists of the historic observations of GDP, population, ARPU, MoU and penetration. The output (white area) consists of historic observations of price and traffic and provides all forecasts. A status column (left) enables users to add or hide observations as new data becomes available, to remove suspected outliers or to try ex-post forecasting.

All calculations fit on this sheet and the results are obtained momentarily by clicking the calculation button. Possible anomalies like eg. few valid observations are highlighted in red.

Market	Model	R^2	$\hat{K}$	$-a$	*t*-rat.	*b*	*t*-rat.	Val.	Rej.
Canada	B	1.00	2.84	0.09	2.52	1.94	48.9	10	6
United States	A	1.00	2.43	1.43	42.5	0.45	6.69	12	0
Argentina	B	0.98	3.02	0.74	1.60	0.63	4.22	11	0
Brazil	A	0.99	7.02	0.30	1.46	0.74	23.1	11	0
Colombia	B	0.98	3.25	0.77	4.04	0.62	9.63	11	0
Chile	B	1.00	3.45	0.45	1.28	0.75	5.50	11	0
Mexico	B	0.99	3.48	0.69	0.96	0.68	2.68	11	0
China	A	1.00	6.41	0.93	10.7	0.56	6.68	12	0
India	B	1.00	4.55	0.62	6.53	0.86	19.4	11	0
Japan	B	1.00	8.31	0.74	5.17	0.67	6.62	11	0
Korea	A	1.00	5.40	0.91	18.4	0.88	19.6	9	0
Malaysia	A	1.00	6.05	0.64	4.17	0.96	9.37	11	0
Taiwan	A	1.00	6.02	0.70	6.85	0.89	23.5	11	0
Thailand	A	1.00	6.29	0.60	4.60	0.91	17.2	9	0
France	B	1.00	1.28	1.12	3.71	0.56	3.82	10	0
Germany	A	1.00	3.17	1.19	5.12	0.47	6.18	10	0
Italy	B	1.00	2.42	0.82	2.11	0.64	4.70	10	0
Netherlands	B	0.99	2.49	0.62	1.08	0.72	2.79	10	0
Norway	B	0.98	0.02	0.35	1.67	1.12	7.62	9	0
Spain	A	1.00	3.70	1.14	13.0	0.78	21.9	10	0
Sweden	A	1.00	7.80	0.39	16.3	1.16	55.2	11	0
United Kingdom	A	1.00	3.47	1.20	7.67	0.76	7.85	10	0
Hungary	A	1.00	5.68	0.55	5.88	0.63	18.9	11	0
Israel	A	0.99	7.57	0.50	2.60	0.76	23.0	10	0
Poland	A	1.00	6.60	0.22	2.64	0.85	30.7	11	0
Russia	B	1.00	4.23	0.24	6.00	0.80	74.9	11	0
South Africa	B	0.99	5.21	0.26	0.40	0.59	5.14	9	1
Turkey	B	0.99	1.95	0.63	3.06	0.92	11.2	9	0

Table 2 Results from fitting the traffic model

7.1 Market Models

The best results from fitting the coefficients as suggested by Eq. (14) – Eq. (16) and Eq. (20) – Eq. (22) to all M markets are given in Table 1 and Table 2 respectively. The tables give the best model and, for this model, the adjusted R^2 value, the fitted coefficients a, b and possibly c with t-ratios, and the number of available and rejected data points for each market respectively.

Table 1 shows that the R^2 values are very close to unit which suggests a high degree of explanation. Moreover, most t-ratios are high which means that the uncertainty of the coefficients tends to be small and that the coefficients typically are significantly distinct from zero. Finally it is noted that six observations were rejected for China, five for India, three observations for Japan, Spain and Poland, two observations for Taiwan and one observation for Turkey.

Table 2 also shows very high R^2 values, which again suggests a high degree of explanation for all markets. Moreover, most t-ratios are even higher, which means that the coefficients tend to be even less uncertain and even more distinct from zero. It may appear odd that all markets contain one more observation than before, but this is because no lagged values are required. Finally it is noted that six observations were rejected for Canada and one observation for South Africa.

The above results suggest that the models work very well in *explaining* the observed evolutions, although it must be noted that the high values of R^2 also can be attributed to the fact that both penetration and traffic are dependent variables. (To see this, note that an observation year $y + 1$ of penetration or traffic can be (recursively) decomposed into a dependent part, the observation year y, and an independent part, the increase from year y to year $y + 1$.)

When it comes to *forecasting*, the true test of the models is to perform ex-post forecasting. To this end, the last two observations were removed from the data sets (one observation was removed for data sets of less than eight observations) after which the models were refitted. The new coefficients and the actual input values C and P were then used to compute forecasts.

The results for all markets are given in Table 3. The table gives the average absolute errors in percent.

It is seen that the differences between the actual and forecast values typically are in the order of 10 % or less. The clear exceptions to this are Argentina and Colombia. We believe that the larger errors for these markets, at least to some extent, are caused by invalid data due to eg. changed reporting, changed coverage or changed regulations as discussed above. It may thus be possible to improve the quality of these (and other) forecasts by rejecting a number of old (obsolete) observations. Such removals can be done manually if eg. detailed knowledge is at hand, or by an algorithm which optimises eg. the adjusted R^2 values or the t-ratios. We emphasise that the second approach requires great care not to 'fit the data to the model' (rather than the other way round).

7.2 Market Forecasts

The models of Eq. (14) and Eq. (16) were fitted as above and used to produce forecasts as suggested by these equations and by Eq. (11) – (13). Forecasts were made for three different cost evolution scenarios.

- One where the annual price drop amounts to 10 % ('Faster')
- One where the annual price drop amounts to 5 % ('Basic')

	Penetration		Traffic	
Market	Val.	Error	Val.	Error
Canada	2	4.65	1	0.14
United States	2	4.32	2	8.01
Argentina	2	26.2	2	22.2
Brazil	2	6.00	3	2.76
Colombia	1	24.4	2	39.5
Chile	2	4.19	2	19.6
Mexico	2	285	2	18.9
China	1	0.83	2	1.40
India	1	0.36	2	7.02
Japan	1	0.07	3	5.28
Korea	2	3.30	2	0.94
Malaysia	2	17.7	2	3.04
Taiwan	2	18.1	2	7.67
Thailand	1	14.2	2	10.3
France	2	0.77	2	1.34
Germany	1	2.94	2	5.27
Italy	2	3.63	2	4.33
Netherlands	2	7.88	2	1.65
Norway	1	0.34	1	1.32
Spain	1	0.28	2	7.45
Sweden	1	0.18	2	2.10
United Kingdom	2	3.47	2	3.44
Hungary	2	9.48	2	8.22
Israel	2	18.3	2	1.58
Poland	1	0.02	2	7.03
Russia	2	12.4	2	7.70
South Africa	2	10.0	2	2.03
Turkey	2	3.22	2	6.46

Table 3 Results from ex-post forecasting

	Penetration				Traffic			
	Optimal		Common		Optimal		Common	
Market	Model	R^2	Model	R^2	Model	R^2	Model	R^2
United States	A	0.9988	A	0.9988	A	0.9991	A	0.9991
China	B	0.9998	A	0.9983	A	0.9987	A	0.9987
Germany	A′	0.9966	A	0.9871	A	0.9977	A	0.9977

Table 4 Goodness of fit for optimal models and common models of penetration and traffic for the three markets

- One where the annual price drop amounts to 0 % (‘Slower’)

It must be noted that the percentages above refer to nominal prices. This means that in real terms there is an additional price drop corresponding to the inflation and the growth of the GDP (which is set to the historic average for each market respectively).

The results for three markets, viz. United States, China and Germany are given below. All numbers are indexed using year 2005 and index 100 as reference points. The markets were chosen because they are big and because they are different. Again it is emphasised that the markets and the numbers are provided by the author only as examples and that they do not, in any way, represent official business outlooks.

We have used common models for all markets in order to simplify comparisons. Applying different models to the same market typically results in similar goodness of fit, see Table 4, but may result in different forecasts. Forecasters must therefore run several models, compare the results and use possible additional information combined with personal judgement before a final forecast is prepared.

Market	United States	China	Germany
Annual price drop	22.5 %	26.8 %	7.99 %
Annual GDP growth	2.12 %	7.24 %	1.26 %

Table 5 Historical characteristics 1995 – 2005 of the three markets

Similarly, we have used common scenarios to simplify comparisons between markets. In reality, forecasters must prepare the scenarios according to local conditions with respect to eg. the degree of competition and the state of the economy. It is interesting to note that the historical price drop as well as the GDP growth differs considerably between the markets, see Table 5.

Finally we have not pruned or corrected data manually. In reality, forecasters must examine all data and judge its relevance and correctness.

The forecasts for *penetration* are given in Figure 2 and we note a stable but weak growth in the United States, a stable and strong growth in China, and a cost dependent growth in Germany.

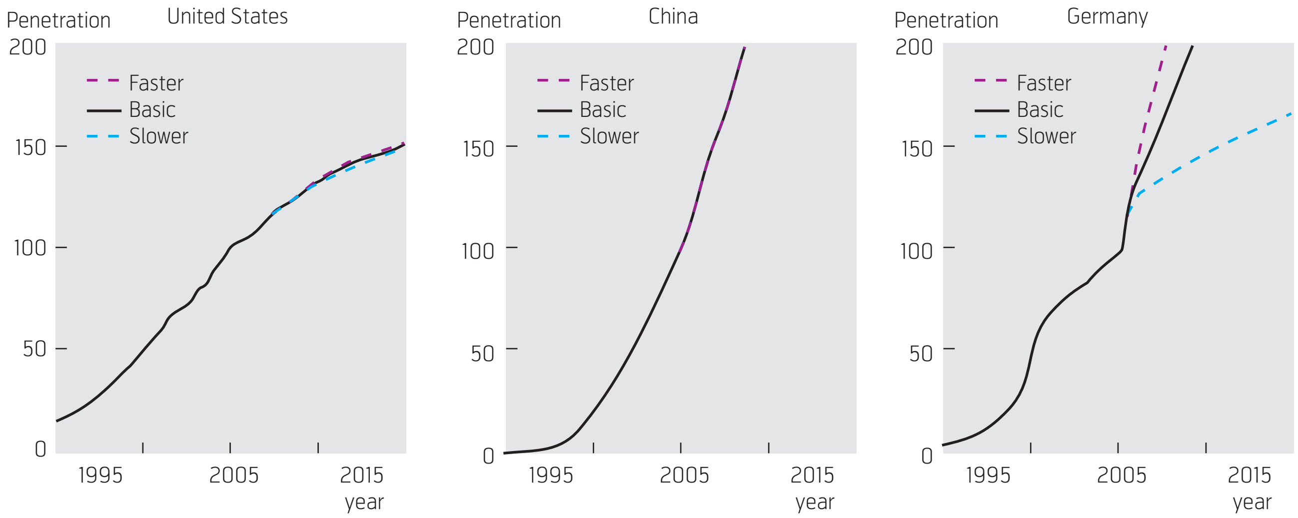

Figure 2 Normalised penetration forecasts for the three markets using model A in Eq. (8). Index 100 refers to 2005

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Market	United States	China	Germany
Cost effect	0.16	0.56	2.55
Network effect	1.54	3.42	1.32

Table 6 Penetration growth characteristics for the three markets using model A in Eq. (10)

To interpret the diagrams we note that penetration is driven by costs (cost effect) and penetration (network effect). The strength of the cost effect can be characterised by the cost sensitivity in Eq. (8),

$$-\frac{\frac{\partial P_y}{\partial C_y}}{P_y} = -a, \tag{26}$$

and the strength of the network effect can be characterised by the normalised remaining market at the current cost, ie. the ratio between an imaginary limit for the penetration and its current value at the current cost. The limit P_∞ with respect to the cost at year y is defined in Eq. (8) as

$$\ln P_\infty(y) = K + aC_y + b \ln P_\infty(y) \tag{27}$$

which gives

$$\ln P_\infty(y) = \frac{K + aC_y}{1 - b}. \tag{28}$$

The numerical values for the three markets are given in Table 6. Comparing the values to the diagrams in Figure 2, we note that

- For the United States, Table 6 shows the weakest cost effect and a medium network effect. Figure 2 confirms that cost has a minor impact (the scenarios tend to overlap) and that there is a moderate network effect (the penetration increases gently). It may be concluded that price cuts will have little influence on penetration.

- For China, Table 6 shows a medium cost effect and the strongest network effect. Figure 2 confirms that cost has a negligible impact (the scenarios tend to overlap) and that there is a strong network effect (the penetration increases sharply). Again it may be concluded that price cuts will have little influence on penetration.

- For Germany, Table 6 shows the strongest cost effect and the weakest network effect. Figure 2 confirms that cost has a significant impact (the scenarios differ considerably) and that there is a weaker network effect (the penetration increases gently without price cuts). It may be concluded that price cuts will have significant impact on penetration.

These findings may not be surprising considering that prices, as defined in Eq. (23), have fallen from one to 0.09 in the United States, to 0.03 in China but only to 0.37 in Germany. Another factor is, of course, the penetration relative to the population, which is about 60 % in the United States, 30 % in China and 90 % in Germany. (Note, however, that ratios above unit already are seen in eg. Sweden.)

The forecasts for traffic are given in Figure 3 and we note relatively similar growth on all markets.

To interpret the diagrams we note that price drives traffic in two ways, viz. directly by increased traffic from existing subscribers (cost effect) and indirectly by additional traffic from additional subscribers (diffusion and adoption effects). The strength of the direct impact can be characterised by the cost elasticity –*a* in Eq. (10). The strength of the indirect impact

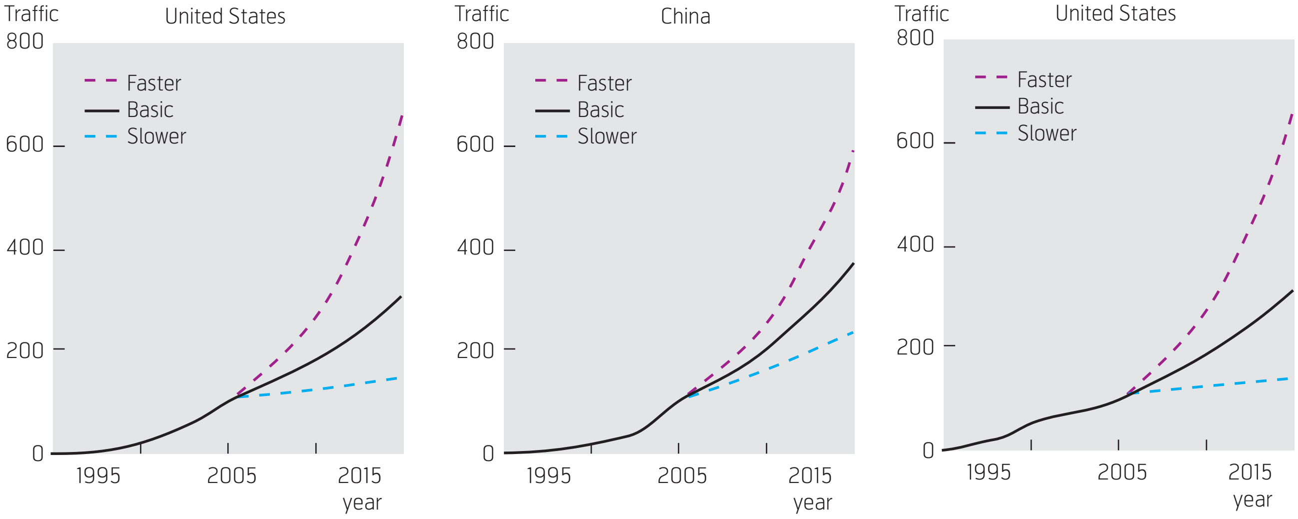

Figure 3 Normalised traffic forecasts for the three markets using model A in Eq. (10). Index 100 refers to 2005

can be characterised by the sum of the penetration elasticity *b* in Eq. (10) and the cost sensitivity of penetration –*a* in Eq. (26). The numerical values for the three markets are given in Table 7.

Comparing the values to the diagrams in Figure 3, we note that

- For the United States, Table 7 shows the strongest direct effect and the weakest indirect effect; hence the increase we see in Figure 3 mainly refers to existing subscribers.

- For China, Table 7 shows the weakest direct effect and a medium indirect effect; hence the increase we see in Figure 3 refers to both types of subscribers.

- For Germany, Table 7 shows a medium direct effect and the strongest indirect effect; hence the increase we see in Figure 3 mainly refers to additional subscribers.

The corresponding evolution of the *minutes of use* as defined in Eq. (11) is given in Figure 4 and we note that the difference between the scenarios is large in the United States, medium in China and small in Germany.

To interpret the diagrams we recall that price cuts will have two effects. Firstly, lower prices will increase the traffic from existing subscribers. Secondly, lower prices will attract additional subscribers. The key point is that an average additional subscriber tends to generate less traffic than an average existing subscriber. The first phenomenon will thus increase the minutes of use whereas the second phenomenon will decrease the minutes of use. The net result of a price cut therefore depends on the relative strengths between the two phenomena.

Market	United States	China	Germany
Direct impact	1.43	0.93	1.19
Indirect impact	0.61	1.01	3.02

Table 7 Traffic growth characteristics for the three markets

- For the United States, we have seen that penetration is driven mainly by network effects, and that cost has the strongest direct impact on traffic but the weakest indirect impact. This means that *existing* subscribers generate substantially more traffic (the strongest direct impact) and that *additional* subscribers are few and similar to the existing ones since they sign up for other reasons but cost (slow growth driven by a medium network effect where costs have the weakest indirect impact). These observations suggest that price cuts should lead to major increases in usage and this is confirmed in Figure 4.

- For China, we have again seen that penetration is driven mainly by network effects, and that cost has the weakest direct impact on traffic but a medium indirect impact. This means that *existing* subscribers generate a bit more traffic (the lowest direct impact) and that *additional* subscribers are many and reasonably similar since they mainly sign up for other reasons but cost (strong growth driven by the strongest network effect and a medium indirect impact). These observations suggest that price cuts should lead to moderate increases in usage and this is confirmed in Figure 4.

- For Germany, we have seen that penetration is driven mainly by cost effects, and that cost has a medium direct impact on traffic but the strongest indirect impact. This means that *existing* sub-

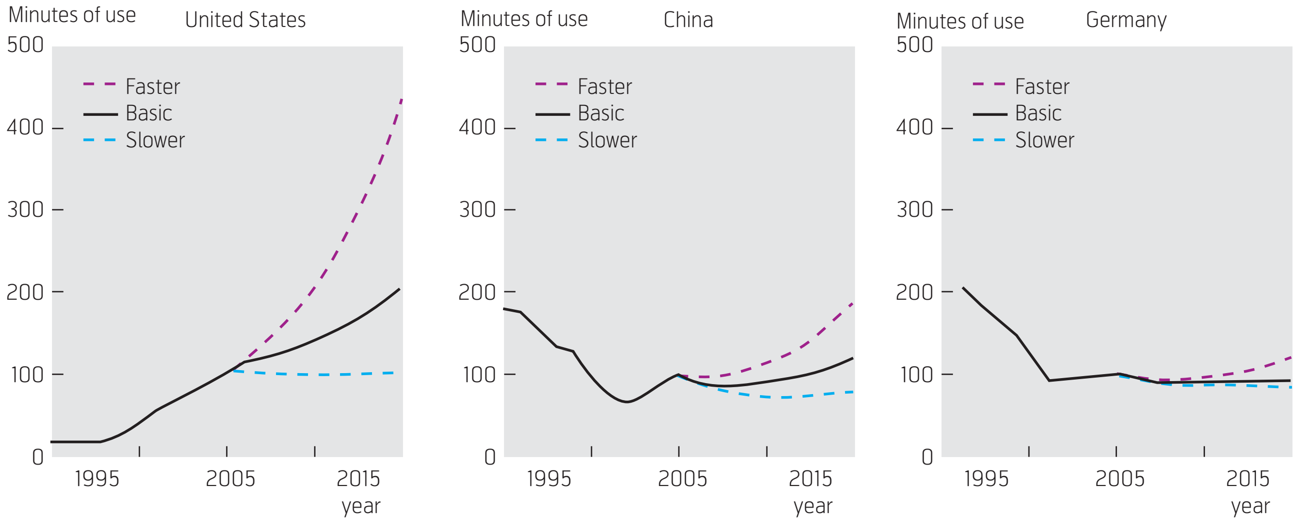

Figure 4 Corresponding normalised evolution of the minutes of use for the three markets. Index 100 refers to 2005

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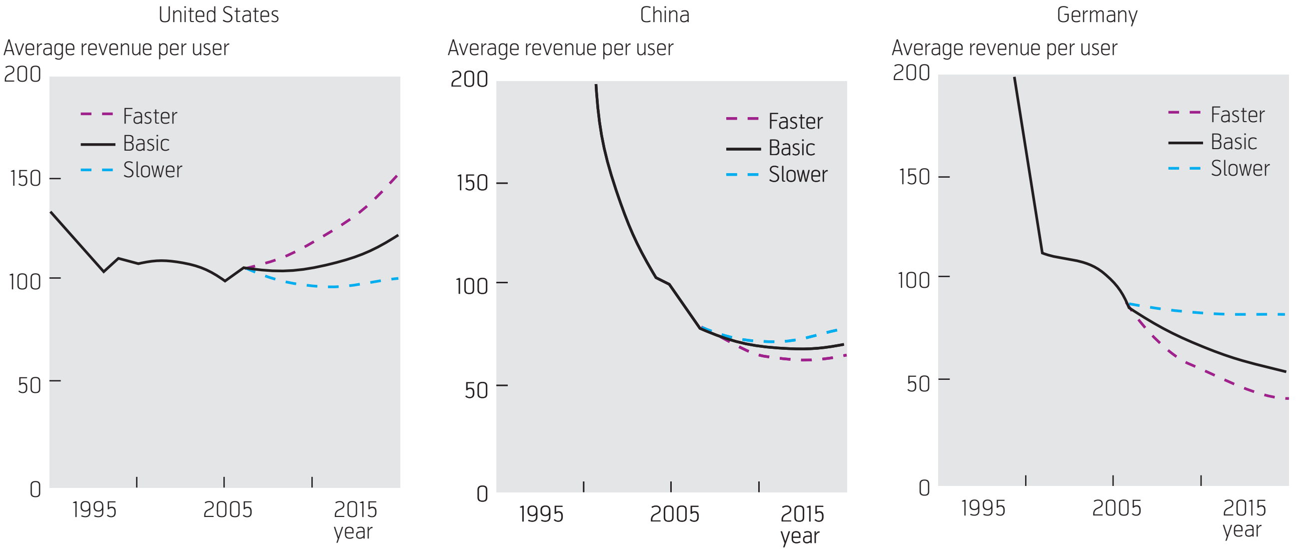

Figure 5 Corresponding normalised evolution of the average (voice) revenue per user for the three markets. Index 100 refers to 2005

scribers generate significantly more traffic (a medium direct impact) and that *additional* subscribers are many and different since they mainly sign up for cost reasons (cost dependent growth driven by the weakest network effect and the strongest indirect impact). These observations suggest that price cuts should lead to minor increases in usage and this is confirmed in Figure 4.

The corresponding evolution of the *average (voice) revenue per user* as defined in Eq. (12) is given in Figure 5 and we note that the difference between the scenarios are large and positive in the United States, small in China and large and negative in Germany. (The reader is reminded that the currency used is the one defined in Eq. (23). This means that revenues are measured relative to GDP such that a constant ARPU essentially means that it increases at the same rate as the economy as a whole etc.)

To interpret the diagrams we again differ between the two effects of price cuts. The first effect, increasing traffic from existing subscribers, will give a positive contribution to ARPU only if the price elasticity of traffic is greater than one. The second effect, additional subscribers with a lower traffic interest, will typically give a negative contribution to ARPU simply because they tend to generate less traffic.

- For the United States, we noted a price elasticity with respect to traffic of 1.4; hence existing users give a large, positive contribution to ARPU, and a small influx of similar additional subscribers (recall that the network effect dominates penetration growth) which, in the worst case, will give a small negative contribution to ARPU. All in all, a large plus and a possible small minus suggest that price cuts may increase ARPU and this is confirmed in Figure 5.

- For China, we noted a price elasticity with respect to traffic of 0.93; hence existing users give a small, negative contribution to ARPU, and a strong influx of reasonably similar additional subscribers (recall that the network effect dominates penetration growth) which, in the worst case, will give a small negative contribution to ARPU. All in all, two small minuses suggest that price cuts will decrease ARPU somewhat and this is confirmed in Figure 5.

- For Germany, we noted a price elasticity with respect to traffic of 1.19; hence existing users give a significant, positive contribution to ARPU, and a strong influx of very different additional subscribers (recall that the cost effect dominates) which, in the worst case, will give a large negative contribution to ARPU. All in all, a small plus and a large minus suggest that price cuts will decrease ARPU significantly and this is confirmed in Figure 5.

The corresponding evolution of the *total voice revenues* as defined in Eq. (13) is given in Figure 6. The reader is again reminded that the currency used is the one defined in Eq. (23). As before, this means that revenues are measured relative to GDP such that a constant revenue essentially means that it increases at the same rate as the economy as a whole etc.

- For the United States, price cuts will push up revenues because of increased spending from existing subscribers.

- For China, price cuts will push down revenues because of decreased spending from existing sub-