Quantitative Networks Analysis and Modeling of Networked Multiagent Environment

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This paper gives an overview of the current research program and some recent results in modelling of modern telecommunication environment obtained in the Department of Industrial Economics and Technology Management, NTNU. We concentrate on different strategic decision problems when it is necessary to take into account uncertainties in demand, technology and other important variables which characterize this rapidly changing environment. Besides, we look at cases that are characterized by the interaction of different agents engaged in relations of competition and collaboration. One such example deals with a quantitative evaluation of business models for collaborative provision of advanced mobile data services while another looks at the relations between network operators and service providers.

Besides, we put our research focus in a wider perspective by presenting a survey of several promising modelling approaches of quantitative network analysis and formation. These include stochastic optimization, statistical mechanics of networks, network formation games and agent based computational economics. All of these research domains study complex interactive systems that are either explicit networks or can be described as such. The paper explores similarities, dissimilarities and linkages of these concepts including approaches on the border between these methodologies. Finally, implications for the research in telecommunications will be given.

1 Introduction

Modern telecommunications environment represents a challenge for industrial planners and academics alike. Only a few decades ago it was a rigidly controlled and static monopolistic environment with a handful of mature services with long life cycles, simple business models and predictable decision outcomes. Now we have a totally different picture of a dynamic industrial reality with a multitude of actors assuming a wide variety of roles, rapidly changing technological solutions, innovative business models, increased uncertainty and risk.

In our research program we focus on the development of methodological tools for an adequate treatment of risk and uncertainty in order to support strategic decisions and the evaluation of business models in telecommunications and the information industry. In this paper we present some of our recent results in this direction. Two examples are described in some detail. The first deals with the modeling of relations between network operators and service providers or virtual network operators with the aim to produce advice for a network operator on his policies towards these actors. The second example deals with the evaluation of service platforms and business models for collaborative service provision of advanced mobile data services. The aim here is to design business models which would induce independent agents to contribute their expertise towards the creation of a successful service or successful service platform.

Both cases deal with situations with considerable uncertainty and risk where several independent agents possess incomplete information about environment and each other's aims. For their analysis we employ the modern modeling methodologies for risk management and optimal decision support under uncertainty developed in operations research and investment science. More specifically, we develop stochastic programming models with bi-level structure enhanced with certain notions of game theory and modern quantitative finance.

In order to put our methodological choices in a perspective we also present a survey of different relevant quantitative methodologies for modeling networks and explore their similarities, dissimilarities and linkages. The first of them is *stochastic optimization* which is specifically developed for the support of optimal decisions under uncertainty. We present an example of an application of this methodology to the planning of service provision. The next two domains are *social network analysis* and *statistical mechanics of networks*. The first evolved in social sciences while the latter has its origins in natural sciences. Key issues are the mathematical description of properties of networks and the exploration of principles behind the network generation and evolution that lead to specific network properties. The reader is referred to "introducing network analysis" by Canright/Engø-Monsen in this issue. Another research domain which is referred to as *network formation or link formation games* applies game theoretic concepts to the analysis

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and formation of networks, where self-interested economic entities build interconnections. Contrary to social network analysis and statistical network mechanics the network generating process focuses on the concepts like Nash-equilibrium, efficiency, and stability. As far as methods of simulation are applied in network formation games this research area intersects with *agent based computational economics*, where the focus lies on the economic network simulation. This approach allows departures from the traditional game theory like imperfections of markets and bounded rationality of the interacting agents. Yet another area of network analysis contains *network design* problems. Here the focus is set on managing physical or material networks in telecommunications, energy transmission, transportation, and others. The key concepts for analysing or evaluating networks are profitability and efficiency. When other concepts like the reliability of data transfer in case of breakdown of links, or the security of data against malicious attacks are introduced then the network design problems find common ground with the other network analysis methodologies mentioned above.

The remainder of this paper will be organized as follows. Section 2 contains a survey and comparative analysis of the network modeling methodologies mentioned above. An overview of our research results with two modeling examples of relations between network operators and service providers and of collaborative service provision is contained in Section 3. The paper concludes with a summary, acknowledgement and an extensive list of the literature.

2 Methodologies for Quantitative Network Analysis

In this section we survey different modern methodologies for network analysis and prepare the ground for a discussion of our current research in Section 3.

2.1 Stochastic Optimization

2.1.1 Introductory Comments on Stochastic Optimization in Telecommunications

Telecommunications has a long tradition concerning the application of advanced mathematical modeling methods. Besides being a consumer of mathematical modeling, telecommunications provided a motivation for the development of areas of applied mathematics. Important chapters of the theory of random processes have their roots in the work of telecommunication engineers. So far this mutual influence was mainly limited to the queuing theory and the theory of Markov processes, but now new decision problems arise which require the application of optimization methods. The recent trends in telecommunications have led to considerable increase in the level of uncertainty which became persistent and multi-faceted. The decision support methodologies which provide adequate treatment of uncertainty are becoming particularly relevant for telecommunications. Here stochastic optimization is the methodology of choice for optimal decision support under uncertainty; see [6],[14],[25]. We start by defining a classification which will serve as a roadmap for the exposition. This classification is made according to the scale of the decision, its relevance within the telecommunications value chain, and the types of uncertainty to be controlled. Besides, different types of uncertainty come into play at different levels. We distinguish three scale levels: *technological*, *network*, and *enterprise* shown in Figure 1. The technological level corresponds to the smallest scale and the enterprise level to the largest and the most aggregated scale.

The *technological level* deals with the design of different elements of telecommunication networks, including switches, routers, multiplexers. Uncertainty on this level is a salient feature of communication requests and flows in the network. Besides, it can arise due to equipment failures. The key decisions are the engineering decisions which define the design for blueprints of these elements. Such blueprints depend on a number of parameters which should be chosen from the point of view of performance and quality of service. Traditionally, performance evaluation of the elements of telecommunication networks was the domain of queuing theory [34]. To be successful the methods of this theory require a specific probabilistic description of the stochastic processes which govern the behavior of communication flows. Usually such a description is not available for new data services, and when it exists, it does not satisfy the requirements of the queuing theory. Stochastic optimization may help to obtain the performance estimates in the cases when

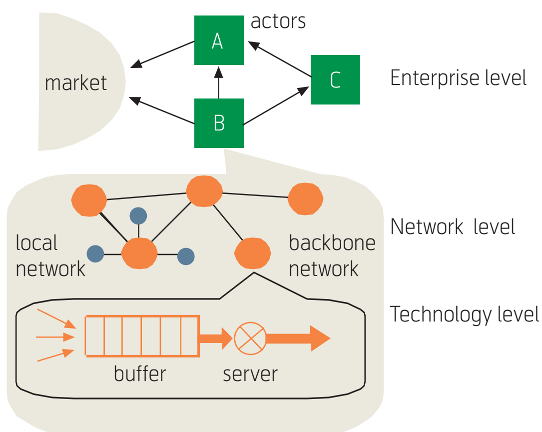

Figure 1 Three modeling levels of telecommunications environment

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more traditional methods are difficult to apply. See Gaivoronski [18] for one such example.

Network level problems deal with the design and planning of different kinds of networks. The application of stochastic programming on the network level will be discussed in more detail in the next section. Section 2.1.3 also contains an example of a stochastic optimization model for design problems at the network level. For related examples, see [7][40][44][15][13][50][2].

Finally, the *enterprise level* is the highest level of aggregation and looks at the telecommunication enterprise as a member of a larger industrial environment which includes other industrial actors and different consumer types. Decisions involve the selection of the range of services which the enterprise will provide to the market, strategic investment decisions, and pricing policy. Market acceptance of services, innovation process and actions of competition constitute the sources of uncertainty which are not present at the lower levels. Telecommunications and, more generally, the information industry differs in important ways from traditional industries due to the rapid pace of innovation. This leads to the absence of perfect markets and to fundamental non-stationarity which makes it difficult to apply traditional micro-economic approaches based on equilibrium. Stochastic programming models enriched with selected notions of game theory can provide more adequate decision recommendations here. We outline one such model in section 3.1. There is no rigid boundary between various levels since decisions made at each level influence decisions on other levels.

2.1.2 Stochastic Programming for Physical or Material Network Design Problems

Network design issues arise in a variety of industries like for aviation [9], shipping [12], water distribution [39], energy distribution [11], and telecommunications [2][18][29][40], and similar problems. In a simplified manner a network design problem can be described as follows. In different geographic locations demand for or supply of commodities or services can be observed. The network has to be designed such that the supply of the service and demand for it are aligned to each other, i.e. a path must exist from the supplying nodes to the nodes where the demand for the services and commodities occurs. The demand depends on the price of the service. Besides, the demand may not be completely specified for a given price but is subject to uncertainty. Furthermore uncertain events may cause components of the network to break down, such that the transportation or distribution needs to take another path through the network if such a path is available. The network designer is confronted with the decisions of pricing, installing links between nodes and routing the traffic through the network. However, the installation, expansion and maintenance of links and capacity as well as processing and transport of data or commodities are costly. Furthermore if the service delivery must be rejected, penalty costs may be incurred. This is especially the case if network failures are observed or if the demand has been wrongly anticipated. The designer seeks for the optimal network design with respect to the expected return on investment, the expected total costs or another equivalent objective.

In telecommunications, networks differ by scale, purpose, and technology involved. We find access networks, local area networks, fixed or mobile networks, and voice or data networks. The decisions involve the placement of processing and link capacities provided by a given technology in a given geographic area with the aim to satisfy aggregated demand for telecommunication services from different user groups. Decisions are often of a dynamic nature and include several time periods. The main uncertainty here is related to the demand for telecommunication services. Due to quantitative and qualitative explosion of such services, this kind of uncertainty increased considerably during the last decade. There are important additional sources of uncertainty connected with possible network failures and future technology development. Stochastic programming methods provide an added value of identifying the robust network design which within reasonable bounds will accommodate the future demand variations. This is particularly true for stochastic programming problems with recourse and multi-period stochastic programming problems which provide intelligent means for mediation between different and often conflicting scenarios of the future. While traditional design approaches are centered on the minimization of the network costs under technological and quality of service constraints, a systematic application of stochastic programming techniques includes the incorporation of modern tools from corporate finance like the evaluation of real options. Comprehensive models which include pricing decisions and binary variables provide a motivation for further development of this methodology.
In the following section we illustrate the general considerations outlined above by one simple yet typical example of the application of the stochastic programming methodology to network planning under uncertainty.

2.1.3 Example: Planning of Internet-based Information Service

We consider here the deployment of an Internet based information service on some territory like a country or a region. The service provider on behalf of which the problem is solved can be the network owner, but can also be a virtual service provider which does not possess its own network and leases network from some network owner. We assume that the network itself exists already and that the decision consists in the deployment of servers at the nodes of this network and the assignment of demand generated in different geographical locations to these servers. More particularly we consider a phased introduction of a service where the deployment in phase 1 with unknown future demand is followed by further deployment in phase 2 that is contingent to the trends in the market. The decisions in the latter phase depend on the project profitability which in turn depends on various options embedded in it, e.g. the option to expand, to abandon and to upgrade the technology. Among various aspects of the problem one can also consider the geographical dimension, the uncertainty of demand and costs, the cost structure which includes fixed and variable costs, the competition and substitution between services as well as relations between different market actors, e.g. network providers and service providers.

In the following we present two steps of the model development. Step 1 represents the simplest possible deterministic planning model which assumes the total knowledge of the market and its future development. Step 2 shows how this model with the help of stochastic programming can be transformed into a more adequate model which takes into account the possibilities to adapt to market reactions and to newly available information.

Step 1: Single Period Deterministic Cost Minimization Model

We start by considering only one decision period and full knowledge about demand and other important parameters. Although these assumptions are highly unrealistic, the resulting model sets the stage for more realistic models. In this setting we assume that the deployment program has to satisfy the known demand fully. The service price is assumed to be given such that the revenues become fixed. For this reason the only way of influencing the profit is by minimizing the costs. Let us introduce some notations.

Notations

$i = 1, ..., n$ – index for regions which constitute a territory where a user population generates demand,

$j = 1, ..., m$ – index for possible server locations,

y_j – binary variable which takes the value 1 if the decision is made to place a server at location j, and 0 otherwise,

x_{ij} – amount of demand from region i served by server placed in location j,

f_j – fixed costs for setting up a server in location j,

c_{ij} – costs for serving one unit of demand from region i by server at location j,

d_i – demand generated at region i,

g_j – capacity of server placed at location j.

Model 1. Find the server deployment program $y = (y_1, ..., y_m)$ and assignment of user groups to servers $x = \{x_{ij}\}$, $i = 1, ..., n$, $j = 1, ..., m$ as solution of the problem

$$\min_{x,y} \sum_{j=1}^{m} f_j y_j + \sum_{j=1}^{m}\sum_{i=1}^{n} c_{ij} x_{ij},$$

$$\sum_{j=1}^{m} x_{ij} \geq d_i \text{ for } i = 1, ..., n,$$

$$\sum_{i=1}^{n} x_{ij} \leq g_j y_j \text{ for } j = 1, ..., m,$$

where y_j takes values from $\{0, 1\}$ and $x_{ij} \geq 0$. Here the first term in the objective function from the first line represents the fixed costs of the deployment of servers while the second term represents the variable costs for serving demand. The objective function is followed by two groups of constraints. The first group is imposed in order to obtain full demand satisfaction, while the second group shown on the last line contains the capacity constraints. This is a well known facility location model and it will serve as a starting point for developing a stochastic programming model with different scenarios of the future demand and a larger number of deployment phases.

Step 2: Two Period Stochastic Cost Minimization Model

We use the previous model as a building block for creating a more adequate stochastic optimization model which takes into account the key uncertainties of the problem. There are several such uncertainties, and most important here is the uncertain user demand. A natural way to describe this uncertainty is the formulation of several *scenarios* about the future demand development. These scenarios can be obtained from market analysis of similar services and expert estimates. In the simplest case we may think about average, optimistic and pessimistic demand scenarios. Each such scenario is described by the

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value of the demand in different regions and by the probability of this scenario.

Two deployment phases are considered: present Phase 1 with known demand, and future Phase 2 with uncertain demand which is described by a finite number of scenarios. The Phase 2 decisions include additional deployment of servers and reassignment of demand to servers in response to the demand development. The decision made during Phase 1 strikes a tradeoff between the minimization of immediate deployment costs and the minimization of average anticipated costs on Phase 2 for additional deployment when demand becomes known. The model follows the framework of stochastic programming with recourse [6]. The formal description of the model is as follows.

Additional notations

$r = 1, ..., R$ – index for demand scenarios,

d_i^r – demand generated by region i under scenario r,

p^r – probability of scenario r,

z_j^r – binary variable which takes the value 1 if under scenario r the decision is made to place a server at location j, and 0 otherwise,

x_{ij}^r – amount of demand from region i served by a server placed in location j under scenario r,

α – coefficient for discounting of the Phase 2 costs to the present.

Each scenario is characterized by a pair (d^r, p^r) where $d^r = (d_1^r, ..., d_n^r)$.

Model 2. Find the Phase 1 server deployment program $y = (y_1, ..., y_m)$, and assignment of user groups to servers $x = \{x_{ij}\}$, $i = 1, ..., n$, $j = 1, ..., m$, as the solution of

$$\min_{x,y} \sum_{j=1}^{m} f_j y_j + \sum_{j=1}^{m}\sum_{i=1}^{n} c_{ij}x_{ij} + \alpha \sum_{r=1}^{R} p^r Q(r,y)$$

subject to the constraints of Model 1. The third term in the expression above represents discounted costs of the Phase 2 deployment averaged over scenarios. The costs associated with scenario r is $Q(r, y)$ and it depends on the Phase 1 deployment decision y. These costs are obtained from the solution of the *recourse* problem for each scenario r:

$$Q(r,y) = \min_{x^r,z^r} \sum_{j=1}^{m} f_j z_j^r + \sum_{j=1}^{m}\sum_{i=1}^{n} c_{ij}x_{ij}^r,$$

$$\sum_{j=1}^{m} x_{ij}^r \geq d_i^r \text{ for } i = 1, ..., n,$$

$$\sum_{i=1}^{n} x_{ij}^r \leq g_j(y_j + z_j^r) \text{ for } j = 1, ..., m,$$

which is similar to Model 1 and chooses the Phase 2 deployment $z^r = (z_1^r, ..., r_m^r)$ and a new assignment of user groups to servers $x^r = \{x_{ij}^r\}$, $i = 1, ..., n$, $j = 1, ..., m$, according to the minimization of fixed deployment costs and variable service costs for a given scenario r. The modern optimization technology permits to solve it for practically important cases, using a combination of commercial solvers like CPLEX or XPRESS with decomposition techniques.

It is important here to note that the deployment decision obtained from the solution of this problem does not aim at the best deployment for any given scenario. This is because the optimal solution for a fixed scenario can be grossly non-optimal if this given scenario does not materialize. Instead, stochastic programming solution aims at obtaining the *robust* decision which will make adaptation to changing demand patterns less painful. More details of stochastic programming approach for network planning are given in Gaivoronski [18].

Evaluation of investment opportunities, real options

The stochastic programming approach allows embedding the modern notions of financial theory and investment science into the process of evaluation of industrial projects. One such important notion is *real options* which represent flexibilities inherent in telecommunication projects [45]. An example where the real option approach can be utilized is the gradual development of a mobile network where new cells are added contingent to an increase of traffic, as opposed to full scale deployment from the start. While for more traditional industries the evaluation techniques can be similar to the evaluation of financial options, for innovative industries with unique projects such approaches are difficult to apply. Stochastic programming models represent an important tool for real option evaluation in such cases. Let us consider some of the options inherent in the example of the service development from above. Here we deal with options to expand, to upgrade technology, to abandon or to convert a part of the infrastructure.

Option to expand (wait and see option). This option is already imbedded in the model outlined above which contains the possibility to add additional servers during Phase 2 contingent to the market trends. The value of this option is obtained by comparing the solution of this model with the solution of the restricted model where there is no additional deployment during Phase 2.

Option to upgrade technology. This is a valuable option because it can dramatically change the project evaluation, especially in an innovative industry like telecommunications. In order to evaluate this option it is necessary to have a closer look at the ways the technology development can affect various components of the model of our example. Namely, the technology development can lead to a decrease in the fixed costs for server installation and/or an increase in the possible server capacities during phase 2. In this case it is necessary to introduce these features into the definition of the scenarios.

Option to abandon. This is a valuable option when the market reaction is uncertain. If demand does not catch up it is reasonable to cut maintenance costs in the regions where demand is weak and possibly recover part of the fixed costs by selling or leasing the server infrastructure.

Results of one such evaluation are represented in Figure 2. This figure shows the dependence of the project value on the service price charged to customers. Three alternatives are shown in this figure. The first alternative is depicted by the green curve and describes the dependence of the project value on the price in the case when no option to expand and no option to upgrade the technology are considered during Phase 2. The second alternative allows an option to expand, but not an option to upgrade technology and is depicted by the black curve. The third alternative shown with the orange line allows both options during Phase 2.

First of all, one notices the jumps on the curves which are due to the discrete character of the decisions. The objective in all three cases is full demand satisfaction. A small increase in price leads to a small decrease in demand which can make a given server redundant with a corresponding stepwise decrease in fixed costs. Another observation confirms the added value of flexibility provided by the options. The value of the project without options is barely positive even for the best choice of the service price. The project becomes decidedly profitable when the option to expand is allowed. There are two regions of profitability with respect to the service price. The first corresponds to an aggressively low service price designed to stimulate large demand and the second corresponds to a less aggressive behavior with high prices and smaller demand. These profitability regions expand when an additional option to upgrade technology is considered. In the absence of options the model recommends defensive behavior with high pricing, while flexibility imbedded in options allows stimulating demand more aggressively with lower prices.

For further details and additional examples of using of stochastic programming models for finding optimal planning decisions under uncertainty in telecom see Gaivoronski [18].

Interaction of market participants
In the problems presented so far the decisions are made by single decision makers who do not have to take into account the strategic behaviour of other market participants. Price choices, traffic routing decisions, and the network deployment are independent from reactions of customers, suppliers and competitors. In reality, however, we have a variety of interacting and mutually reacting players on different decision layers. One player's decisions affect the other players' strategies, and vice versa. This constellation is considered in the approaches presented below. In particular, these are network equilibrium problems (Section 2.2), network interdiction (Section 2.3) network formation games (Section 2.4) and constellations as described in chapter 3 where interdependencies of several network operators and service providers are modelled.

2.2 Network Equilibrium Problems

Network equilibrium models are commonly used for the analysis and prediction of traffic patterns in transportation, distribution or telecommunication networks where congestion occurs. The reader is referred to Nagurney [36] who gives an outline of the historical beginnings of network equilibrium models. Assuming a given network, the users or applications compete for the given resources. They analyse the state of the network and individually optimize flow and routing from a supplying node to a demanding node. Each application's decision changes the state

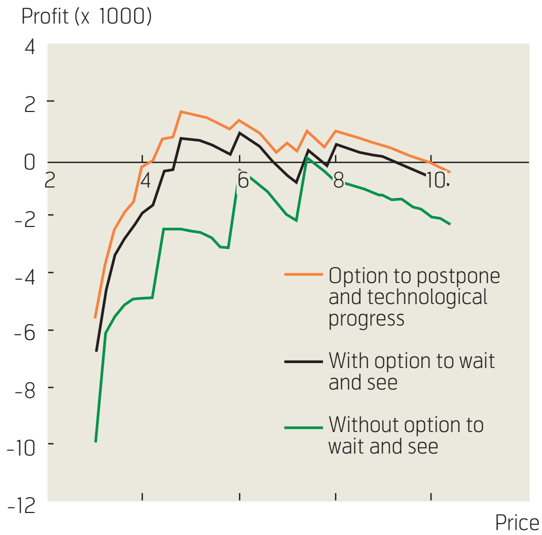

Figure 2 Evaluation of real options in the case of service introduction

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of the network and so do the optimization problems of the other users or applications. In this section we focus on the network design which is finalized before the network usage and the occurrence of equilibrium. Mathematically we have a bi-level planning problem à la von Stackelberg [46]. In the first level the network designer makes his choices to install, upgrade, or abandon parts of the network such that the operator maximizes revenues or social wealth that stems from the usage of the network. In the second stage multiple network users maximize their wealth under the given network topology and under anticipation of the other users' behaviour. The formulation of network design problems as bi-level mathematical programs goes back to LeBlanc [28] (see also [42]) who studies a public highway network which is planned and implemented by the public sector and used by private individuals. While the government attempts to maximize the social welfare, each individual selfishly pursues own interests like minimizing the travel time when moving from point A to point B in the network.

Marcotte [31] represents a network design model where the equilibrium flow problem is formulated as a variational inequality. The objective of the network designer is to minimize the total traversal time and investment costs of the network, while the users optimize individually their flow traversal time.

2.3 Studies of Survivable Network Design or Network Interdiction

Most of the studies in network design, may they be of an optimization approach or with a game theoretic background, are designed from a cost minimizing or profit maximizing perspective. The field of *survivable network design* adds objectives and measures for maintaining a reliable network in case of failures of network components [35]. However, these failures are assumed to be of an accidental and random nature. Targeted attacks of rational agents who aim at a substantial loss of network performance are studied in the so-called approaches of *network interdiction*. For example Smith/Lim/Sudargho [41] consider a three-level, two-player framework, where the first level network designer constructs a network and sends multi-commodity flows through the network. In the second level an attacker attempts to destroy the network performance by destroying links. Three strategies are considered for the attacker: (a) destruction of the links with largest capacities, (b) destruction of the links with highest initial flow, (c) destruction such that the maximum post-interdiction flow is minimized. While (a) and (b) are heuristics of a bounded rational attacker, (c) is the strategy of a rational player.

2.4 Network or Link Formation Games and Games on Networks

A *network formation game* (also referred to as *link formation game*) is given by a set of players where each player decides individually with what other players he/she wants to create links (connections). The formation of a link causes costs that are either carried by the node that initiated the link or will otherwise be shared by both the nodes. In the link creating process each player pursues individual interests, i.e. he weighs the benefits from being directly and indirectly linked against the costs from initiating, installing and maintaining links. The utility that a player can receive depends on his/her own actions as well as the actions of other directly or indirectly connected players. Links may represent friendships, co-authorships, common research projects, trade agreements, political or economic alliances, and others. Models of network formation can be classified as either static (Jackson/Wolinsky [24]) or dynamic (Jackson/Watts [23]). In the first kind of models the issues involved are the following:

a) Which network topology is efficient?

Different concepts of efficiency can be applied. In the case of strong efficiency the total value of the obtained network is higher than the total value of any alternative network structure. In the concept of Pareto efficiency the value of each single player is considered rather than the total value of the network: for a given value function and allocation rule, a network structure is Pareto efficient if no other network structure exists that gives a higher pay-off to at least one agent, without reducing the pay-off of at least one other agent.

b) Which network topology is stable, i.e. does an equilibrium exist for the network game?

An equilibrium is reached if no player has an incentive to unilaterally change its own prevailing linkages to other individuals. Hence, the network structure will come to a resting point. For the treatment of these issues in static settings see Jackson/Wolinsky [24].

c) How is the value allocated to the individuals in equilibrium?

d) What pay-off structures or allocation rules are necessary for the network to become efficient or stable?

The section on network formation in the survey of Kosfeld [27] of network experiments also gives an overview of some interesting applications. Two of the often cited network formation games are the connections and the co-author model (Jackson/Wolinsky [24]). In the connections model social relations between individuals are represented as links. Having both direct and indirect relationships to other individuals incurs benefits which may be in the form of friendship, social integrity, access to information, and others. Direct relationships offer the highest benefit, and the longer the path to another individual, the less this benefit becomes. In the specific case of the symmetric connections model, depending on the parameters of the pay-off function, the unique strongly efficient network is either a star, a complete graph or an empty graph. Here the complete graph is a unique pairwise stable network while the star does not necessarily reflect this property. However, an empty graph is not stable. In the co-author model each node represents a researcher who works on different projects. Links represent the fact of two researchers being involved in the same project. The time that two researchers spend within the project determines their synergy. The more projects one researcher has the less time he can spend within the project; hence the less synergy will occur. In this model strong efficiency occurs if there are separate pairs of authors that are connected, and the pairwise stable network can have fully intra-connected components that vary in size.

Slightly different from network formation games are theoretical and empirical studies with respect to games that are played between individuals in populations (see Kosfeld [27] for a survey). In many of these studies the network structure is given and individuals play games on this particular topology. The purpose here is to evaluate the affect of network structure on how the individuals play games with each other. Then different network topologies can be compared with respect to stability and efficiency of the individual decisions. Phan [37] for example studies the prisoner's dilemma played among individuals on different network constellations. In particular the dominance and transition of strategies are compared for a regular network on the one side and a small world network on the other. The players do not have complete information on the whole network. Each player only observes the pay-offs and strategies of his/her neighbours. The decision rule of an agent is to apply the strategy within his clique (consisting of him and his restricted number of neighbours) that gives the maximum payoff. Furthermore, accidental defection by a certain number of players is introduced symmetrically into the network. The results of Phan show that in regular networks the whole population will tend to defect instead of cooperate, i.e. the welfare of the population is reduced to its minimum. When the small world property is introduced the defection does not necessarily spread over the whole population. Hence, the small world network allows obtaining the higher welfare of the population.

In another study of Goyal/Vega-Redondo (2005) costs for establishing links are introduced. These links are then preconditioned for playing the stage game. In this study conditions for connectedness or emptiness of the network are derived. In this case individual decisions take affect on the composition of the network.

However, many network formation games lead to topologies that are not in alignment with the properties of real world networks found in social network analysis and statistical network mechanics (see "Introducing network analysis" by Canright/Engø-Monsen in this issue). Hence, it remains interesting to investigate what economically driven decision rules and processes result in network topologies observed in practical cases.

2.5 Agent-based Computational Economics and Multiagent Networks

Within agent-based computational economics (ACE) complex agent-based systems are studied by means of computerized simulation. The objective is to analyse the dynamics, global properties and patterns of complex systems (like networks or societies) at the macro-level and analyse their emergence from the autonomous, heterogeneous, individualistic, idiosyncratic, self-interested and interacting behaviour of individuals on the micro-level. ACE follows the traditional studies on self-organizing economies originated by Smith, Hayek and Schumpeter. However, only the recent developments in computational power made ACE possible. The advantage of ACE compared to conventional quantitative modeling of agent-systems is that the agents can have a richer heterogeneous internal cognitive structure. However, departing from traditional game theory the individuals are characterized by bounded or procedural rationality.

Normative recommendations are derived on how the individual actions are successful in complex environments or how mechanisms can be imposed by regulators to take a desired effect on the complex system. Adding to Tesfatsion's extensive internet presentation on this topic (http://www.econ.iastate.edu/tesfatsi/ace.htm, 2006) the survey article by the same author [43] gives an introduction and an overview of several applications of ACE. Tesfatsion [43] addresses various fields of ACE studies, among which the following are of particular interest for the discussion of network formation and analysis: (a) ACE research on learning, (b) bottom up modeling of market processes, and (c) formation of economic networks.

In (a) researchers are motivated to find out how different learning schemes affect the outcome of the simulated system with respect to improved efficiency, global optimality, selection from multiple equilibria, etc. often in contrast to traditional models that presume rational choice as individual behaviour. Learning might be simply imposed or empirically substantiated, and learning schemes may contain self-reflection as well the reflection of other players' strategies (see for example Vriend [47]). One research direction that benefited from the studies of learning within ACE is that of the application of evolutionary algorithms to economic problems. In this area Arifovic [3] gives a survey of research that addresses the following issues: "(1) the convergence and stability of equilibria in the models with unique rational expectations equilibria, (2) the use of the algorithms as equilibrium selection devices in the models with multiple equlibiria, (3) the examination of transitional dynamics that accompanies the equilibrium selection process, (4) examination of learning dynamics that are intrinsically different from the dynamics of the rational-expectations versions of the models." (See Arifovic [3], p. 374.)

Issue (b) addresses the question of how markets organize themselves or how transitions from and to market equilibria take place. This issue is strongly connected to the issue (a) mentioned above, since the market outcome strongly depends on the learning schemes applied by the modeller. This issue is considered in the studies by Balmann et al. [5], who look at the application of a parallel genetic algorithm to an agricultural market problem.

For issue (c), the formation of economic networks, Tesfatsion [43] narrows the research focus with the following questions: "What drives the formation of interaction networks among buyers and sellers? How do these networks evolve over time? What are the social welfare implications of these networks?"

Some studies can be placed at the intersection of statistical network mechanics and ACE. For example Wilhite [48] compares four types of trade networks: (1) completely connected networks, (b) a network of disconnected trade groups, (c) a network of trade groups that are aligned around a ring where one trader of a group is connected to one trader of the neighbour trade group, and (d) small-world networks. The consequences of these network structures for a bilateral trade are studied with respect to the trade-off between market efficiency and transaction costs. Wilhite finds that the small-world trade network provides market-efficiency close to completely connected networks and a reduction of transaction costs as in locally connected networks. He also hypothesizes the existence of micro-level incentives for the evolution of such a network structure, i.e. due to the advantages of a network with small world property, the agents self-organize to such a network type. Other researchers focus even more explicitly on the formation of such networks (see Vriend [47]).

Another ACE approach that uses the results from social network analysis and statistical mechanics of networks is provided by Phan/Pajot/Nadal [38]. Basically, they study regular, random and small world networks of individuals. They study the case of a monopoly that sells a single product to their customers. Customers interact with each other and influence each other's surplus function that each customer maximizes. These network externalities depend on the topology of the network. The surplus function is defined as the idiosyncratic preference for the product plus the social influence through neighbours who also use the product minus the price that needs to be paid for the product. The monopoly's objective is to maximize the profit considering the individual choices of the customers who are affected by their interaction that depends on a certain network structure. The paper shows that the monopoly's price depends on the structure of interaction between customers. Hence, it is recommended for a monopolist to analyse the network structure for deriving optimal decisions. The optimal price and the profit increase with the degree of connectivity and with the range of interaction.

The concept of *agent nets* developed in Gaivoronski [17][19] and Bonatti, Ermoliev and Gaivoronski [8] also belongs to the class of ACE models. In these papers the formal definition of agent nets was developed particularly suited for modeling of industrial relations in the information economy. Based on these ideas the modeling system MODAGENT was developed and used for the analysis of typical constellations of industrial agents in the telecommunications sector.

3 Some Current Research Issues: Competition and Collaboration in the Networked Telecommunication Environment

The previous section gave a broad overview of quantitative models for the analysis of different kinds of networks. Here we give two examples from our current research which utilize some of the methodologies described above for modeling strategic decisions in the telecom market. Both the examples are united by the common objective: provide quantitative models for support of strategic decisions in the situations which are characterized by the following two features:

- Uncertainty about important parameters which influence decisions, like demand, technology, user behaviour, market conditions, etc.;
- Presence of several independent actors who assume different roles and engage in complex relations of competition and collaboration.

Our focus on these two features is due to the observation that they play a more and more important role in advanced industries like telecom or more generally, the information industry compared to more traditional industries. Consequently, from the methodologies presented above we select stochastic programming as an adequate methodology for dealing with complex decisions under uncertainty. It is enhanced by certain concepts borrowed from game theory and network games, a natural choice to represent actors who take independent decisions. In our future research we are planning to expand this analysis by integrating concepts from the agent based computational economics similar to how it was done in Gaivoronski [17][19], and incorporating insights from the statistical mechanics of networks or social network analysis. A promising direction to go is the representation of the market as an interaction system that can be described as a network which shows properties like high clustering, small worlds, and power laws in node degrees. Instead of using traditional aggregated demand functions the market is modelled as a social network and as such builds into the hierarchical decision models of different actors in telecommunications.

For now we look at the following two situations:

Virtual network operators. There are two or more telecom operators who provide a similar service to a population of users. One of these operators, called *network operator* (NO), possesses the entire network infrastructure to provide this service, while others, called *virtual network operators* (VNO), do not operate the network themselves. They need to lease the network capacity from the network operator to provide their service. There is a lot of uncertainty in this environment, including market projections, user response, and mutual knowledge of the operators about parameters of their respective business models.

We develop a model that allows answering the following questions: What are the market conditions under which this relationship will be mutually beneficial? When will all operators continue to offer a service, and when will some of them have to exit from the service provision? What are the responsible bounds that a regulator can impose on the leasing prices? What is the pricing scheme for virtual operators to bear a fair share of the costs for maintaining and developing the network infrastructure?

Provision of advanced mobile data services. Provision of such services involves concerted effort of many actors which assume different roles in the service provision. Some of them will contribute with network capabilities, others with content, still others with organizational effort like brokering or billing. They are all independent actors pursuing their business objectives, and yet they should decide to unite their efforts if a service is to come into being. Services are united in bundles or platforms and they compete between themselves and with traditional services for users' attention. The following issues are addressed: What will distinguish successful services or service platforms from unsuccessful ones in such a dynamic and uncertain environment? Which traditional and new business models should be adopted for service provision? What roles can the actors combine and which combinations are detrimental for the business? We will try to answer these questions by drawing upon developments in stochastic programming and ideas from modern finance and investment science.

3.1 Virtual Network Operators

We use this example to describe a modeling approach for the provision of decision support and strategy evaluation of an industrial agent in complex relations of competition and collaboration with other agents in the telecommunication environment. This is the situation of many telecom service providers nowadays, with a deregulation process and convergence between telecommunications, computer industry and content provision being well under way. The objective of the approach is to provide a set of quantitative decision support tools which would enhance the quality of strategic and tactical decisions.

Microeconomic theory [33] provides important theoretical insights into these issues, especially when the studied system is under conditions of equilibrium. However, classical theory often treats uncertainty inadequately. Unfortunately, central features of today's telecommunication environment are the presence of uncertainty and, usually, the absence of equilibria. This makes many established approaches inapplicable. Therefore we employ techniques that are specially designed to incorporate uncertainty and dynamics in decision models, namely approaches and methods related to stochastic programming [6], [14]. On the theoretical level, such techniques have been under development for a few decades, but only relatively recently has the state of software and hardware allowed large scale applications. We supplement this by selected ideas from game theory because a part

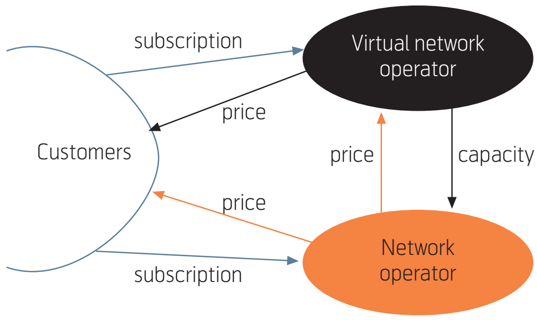

Figure 3 Relations between customers and network operators

of the uncertainty that a given decision maker faces results from actions of other decision makers.

Figure 3 shows relations between service providers and a customer population which we are going to study. The considered time horizon consists of several time periods. We assume that the two operators provide a common market with the same or similar type of service based on the telecommunication network. For delivery of this service they utilize network capacity. Whereas one of the providers owns the network, the other one is a virtual operator without network facilities. The latter needs to lease capacity from the network owner in order to provide the service

Since the aim is to provide decision support tools for a given actor we do not follow the usual economic view on a market "from above", i.e. the maximization of a general welfare [33]. Instead, the point of view of one of the providers is adopted here. His main focus lies on maximizing his own profit or another business performance measure. We take the point of view of the network operator, but the virtual operator could be considered similarly. In order to achieve his goal the network provider formulates predictions of the customer behavior and his rival's responses to his policy. The prediction models depend on a number of parameters with uncertain values, which makes an adequate treatment of uncertainty particularly important.

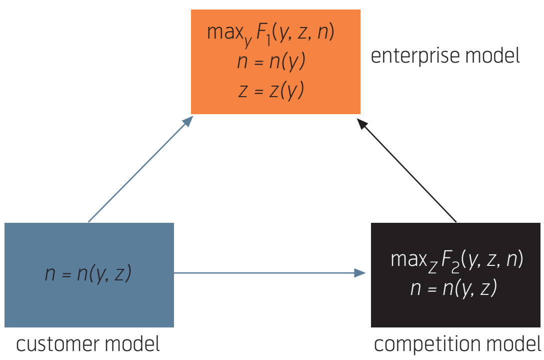

Figure 4 Structure of the model bundle

Following this approach, the decision support model of the network owner consists of a coordinated bundle of submodels: *enterprise model*, *competition model* and *customer model* that are connected as illustrated in Figure 4.

At the beginning of each time period the network operator performs the following steps to determine his optimal decision under the current circumstances:

- Predict the customer response for a given decision and a given competition response using the customer model. This yields the estimate of the customer numbers for both the network operator and the competition.
- Predict the competition response for a given decision using the competition model.
- Select an optimal policy from the enterprise model by using the predictions of the customer and the competition response obtained in the previous two steps.

The following notations are utilized in Figure 4:

y – decisions of the network operator (NO): price y_1 for service provision to customers and price y_2 for capacity leased by his competitors. Besides, the decisions for upgrading and expanding the network capacity can be included here.

z – decisions of the virtual network operator (VNO): price z_1 for service provision and amount z_2 of capacity leased from the NO.

$n = (n_1, n_2)$ – total number of customers of the NO and the VNO respectively. These numbers depend on the respective decisions y and z.

$F_2(y, z, n)$ – performance measure of the VNO like profit, revenue or market share. It depends on both provider's decisions y and z and on the number of his customers $n = n(y, z)$ obtained from the customer model. It comprises the network operator's knowledge about his rival's aims, namely the NO thinks that the VNO chooses his decisions from maximization of this performance measure. More formally, the network operator uses the predicted decision $z(y)$ of the virtual operator which is the solution to the following problem:

$$\max_{z \in Z} F_2(y, z, n(y, z))$$

where Z is the set of admissible decisions of the VNO.

$F_1(y, z, n)$ – performance measure of the NO, which depends on decisions of both providers, y and z, and on the number of his customers $n = n(y, z)$ obtained from the customer model. For a fixed decision y the value of this function is computed using the prediction $z(y)$ of the virtual operator's response and the prediction $n(y) = n(y, z(y))$ of the network owner's customer number. Consequently, the decision y is found by solving the problem:

$$\max_{y \in Y} F_1(y, z(y), n(y))$$

where Y is the set of admissible decisions of the network operator. Both functions are average performance measures where the averages are taken with respect to the values of random parameters which enter the description of the problem, like the customer response to the price change, reciprocal knowledge about the production costs, etc. Besides, the gradual acquisition of information by the actors in a dynamic setting and their response to changing market conditions are also included in the model.

A typical example of this modeling advice is given in Figure 5. It shows how the expected profit of the network operator depends on his pricing decisions y_1 and y_2. The decision space in this example can be divided into four regions:

- *Normal competition.* This regime happens when both service prices and leasing prices are moderate. Both providers are present on the service market and the revenue of the network operator is composed from two parts: service provision and network provision.

- *Network operator service monopoly.* This regime is the result of high leasing prices and moderate service prices. The price of entry to the service market becomes prohibitive and only the network provider develops the service provision capabilities while VNOs stay away.

- *Core business solution.* This is the regime with moderate leasing prices and high service prices. All operators concentrate on their core business, i.e. the network provider maintains and develops the network and leases capacity to VNOs who concentrate on the service provision to customers.

- *Market collapse.* It happens with high leasing prices and high service prices. High leasing prices prohibit the entry of the VNO to the service market while the high service prices scare off the customers. As a result, there is no service provision by any of the operators. Obviously, this regime is to be avoided.

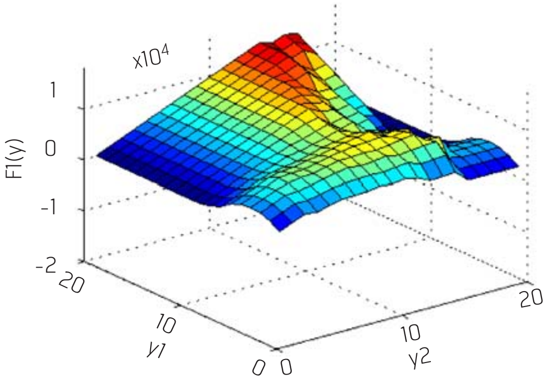

Figure 5 Dependence of profit of the network provider on his decisions

Having this decision support tool, the network operator can decide which regime is more profitable to him or corresponds better to his aims. The network operator also obtains insights into how other business decisions like production cost reductions or technology upgrades will affect his performance, and how his knowledge about competition can affect his strategy.

More details of this example can be found in Audestad/Gaivoronski/Werner [4].

3.2 Provision of Advanced Mobile Data Services

In this section we draw upon the modeling experience of multi-agent environments obtained during the studies of relations between network operators and virtual network operators and enrich it with some modern notions of financial theory and investment science.

General Setting

The design of advanced mobile data services to be carried on 3G networks and beyond is a hot topic in the telecommunication industry and academy. This is because the business success of the provision of such services will define the business success of the mobile operators and other relevant industrial actors in the near to medium future. In this respect considerable attention is given to the design and development of service provision platforms which support a set of tools and basic services that facilitate the development, deployment and customization of specialized services by service providers and even nonprofessional end users. Such platforms are yet to appear in commercial use in the mobile environment, but they already exist on the Internet.

Deployment and operation of service provision platforms and provision of individual services require collaboration of different industrial actors who contribute to the common goal with their individual

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capabilities and expertise. One can think about fixed network operators, mobile operators, providers of different information content, internet providers, software developers and other actors who will join their forces to provide a successful service. This gives a rich picture of a service provision environment where a multitude of actors cooperate and compete in order to deliver a wide range of services to customers in a profitable manner.

Understandably, the main efforts in research and development so far have been concentrated on technological and engineering aspects which enable the provisioning of advanced mobile data services. The history of information technology testifies, however, that the possession of the best technological solution is not necessarily enough to assure the business success of an enterprise. A very important and sometimes neglected aspect is the design and evaluation of an appropriate business model which would support the service provision. Business models for service provision by a single actor are pretty well understood, both organizationally and economically. This is the case, for example, for the provision of the traditional voice service over a fixed network. When an actor evaluates the economic feasibility of entering the provision of such service, he can employ quantitative tools developed by investment science, like the estimation of the Net Present Value of such a project [30]. Usually an actor should choose between several service provisioning projects, each characterized by return on investment and the risk involved. Then the portfolio theory [32] suggests a way to balance between return and risk and to select the best portfolio of projects taking into account the actor's risk attitudes. An adequate risk management is especially important in a highly volatile telecommunication environment. Industrial standards in this respect are starting to emerge, originating from the financial industry [1]. Industrial projects in high-tech industries are often characterized by considerable uncertainty and at the same time carry different flexibilities.

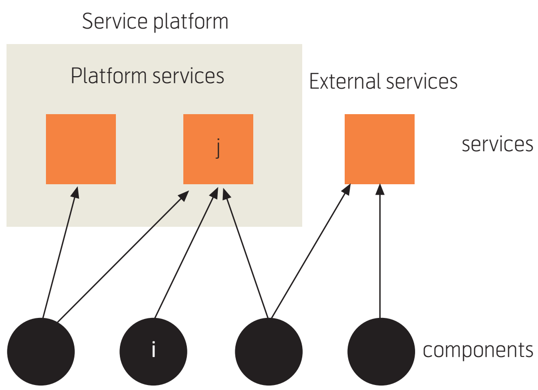

Figure 6 Two level service composition

Stochastic programming provides the optimization models for adequate treatment of uncertainty and flexibilities in the planning of service provision.

Business models for cooperative service provision that involve different constellations of actors are studied to much lesser extent and the quantitative analysis similar to what exists for the single actor case remains a challenge. The methods mentioned above are all developed to be used by a single actor engaged in the selection and risk management of his portfolio of industrial projects. The influence of other actors is present only implicitly on the stage of the estimation of the future cash flows. This is not enough for an adequate analysis of collaborative service provision. Suppose, for example, that a service provider delivers a service to a population of users and receives revenues for this delivery. If a service is composed from modules and if the enablers are provided by different actors then the service provider has to decide about the revenue division between these actors such that it becomes attractive for them to participate in the service composition and provision. This revenue sharing decision together with a concept of what is attractive to other actors should be explicitly incorporated into the evaluation of the profitability of this project.

Our aim here is to contribute to the adaptation and further development of the methods of evaluation and risk management of business models and industrial projects for the case of the collaborative service provision. We look at the actors engaging in a service provision as making a decision about the composition of their portfolio of services to which they are going to contribute. They do this independently following the risk management framework of portfolio theory. The pricing and revenue sharing schemes induce the actors to contribute the right amount of provision capacity to participation in the service provision. We develop a two tier modeling framework which results in the optimal selection of pricing and revenue sharing. This is done by utilizing the approach of stochastic optimization with bi-level structure [4].

Let us outline how this methodology coupled with notions of investment science can be used for decision support and evaluation of business models for collaborative service provision.

Model of Service Structure and Provision

The composition of a service can be quite complex. For the purposes of clarity we use here a simplified description which still possesses the main features of the provision environment important for business modeling. Namely, two levels of the service composition will be considered here as shown in Figure 6.

In this case the service environment is composed of two types of services. The first type comprises services whose structure and provision we are interested in and which we are going to consider in some detail. They can be provided in the context of a service platform and therefore they will be referred to as *platform services*. There will also be *external services* whose structure is of no concern to our modeling purposes. They are present in the model in order to adequately model the environment in which the provisioning of the platform services happens.

The main building blocks of the platform services are service components and/or enablers indexed by $i = 1 : N$ and services indexed by $j = 1 : M$. Here and in the rest of the section we shall use the term *components* as a generic term for software components, enablers and enabler services which compose a service. Components are measured in units relevant for their description, like bandwidth, content volume, etc. The relation between components and services is described by coefficients λ_{ij} which measure the amount of component i necessary for provision of the unit amount of service j. Thus, a service j can be described by vector

$$\lambda_j = \lambda_{1j}, ..., \lambda_{Nj}.$$

A service j generates a revenue v_j per unit of service. This quantity depends on the service pricing which in turn depends on the user behavior and market structure. For the moment let us assume that v_j is a random variable with known distribution which can be recovered from expert estimates and from simulation models that explore the structure of user preferences and market features. The random variables v_j can be correlated due to the service substitution, macroeconomic phenomena and other causes.

Services can be provided by different constellations of actors. Here we consider one such constellation where the actors are the enterprises which have the capability to provide service components assuming different roles. Different constellations can be considered in a similar manner. In this section we shall focus on the two tier structure of the service provision.

- *Component provision layer.* For the matter of simplicity we consider generic actors who provide just one specific component for different services. Such *component providers* can correspond to real actors or to business units of real actors if the real actors fulfill several roles and provide several components. The objective of a component provider is to select a portfolio of services to which this actor will make a contribution. This decision is made on the grounds of balance between projected profit from component provision balanced against the risk of variations in demand and service acceptance among the prospective users of services. In order to quantify this decision process it is necessary to use a simplified profit model for an actor.

- *Service provision and platform provision layer.* There is one actor who provides the service aggregation and organizes the overall service delivery to the end users, this actor will be referred to as a *service provider*. This actor can provide the whole bundle of platform services and will decide which services to include in this bundle, and is then called a *platform provider*. He will collect the revenue from the end users and distribute it among the component providers according to some revenue sharing scheme. This scheme is defined by a vector of revenue shares decided by the service provider

  $$\gamma_j = (\gamma_{1j}, ..., \gamma_{Nj}),\ \gamma = (\gamma_{11}, ..., \gamma_{N1}, ..., \gamma_{1M}, ..., \gamma_{NM})$$

  such that an actor that contributes with the component i receives the revenue $\gamma_{ij} v_i$. Determination of these revenue sharing coefficients is one of the objectives of the design of the business model for service provision.

Besides platform services the actors can supply components also to external services. The structure of these services is not specified and it is assumed that they are fully described by the revenue v_{ij} generated by provision of the unit of component i to external service j, $j = M + 1, ..., K$.

Component Provision Layer

Let us describe how the component providers decide to join the provision of a particular service. We assume them to be rational economic agents that pursue the aim of maximizing their profit. They select the services to which their efforts contribute similar to how an enterprise will select its portfolio of industrial projects or how a bank would select the portfolio of financial assets for investment. Therefore the set of services to which a provider of a given component contributes will be called his *service portfolio* and we shall utilize portfolio theory [32] in order to model the composition of his portfolio. Portfolio x_i is defined by shares x_{ij} of the component provision capacity that the component provider i allocates to service j, $j = 1, ..., K$:

$$x_i = (x_{i1}, \ldots, x_{iK}).$$

The next step is to define the revenues, costs, profit and return on costs of the component provider.

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Revenues:

$$V_i = W_i \left(\sum_{j=1}^{M} \nu_j x_{ij} \frac{\gamma_{ij}}{\lambda_{ij}} + \sum_{j=M+1}^{K} \nu_{ij} x_{ij} \right)$$

where W_i is the provision capacity of the provider of component i.

Costs:

$$C_i = c_i W_i$$

where c_i is unit component provision cost. Here we assume that all component provision capacity is utilized and that the fixed provision costs are included in the variable costs.

Profit:

$$\pi_i = W_i c_i \left(\sum_{j=1}^{M} x_{ij} \left(\frac{\nu_j \gamma_{ij}}{c_i \lambda_{ij}} - 1 \right) + \sum_{j=M+1}^{K} x_{ij} \left(\frac{\nu_{ij}}{c_i} - 1 \right) \right)$$

Return on costs:

$$r_i(x_i) = \sum_{j=1}^{M} x_{ij} \left(\frac{\nu_j \gamma_{ij}}{c_i \lambda_{ij}} - 1 \right) + \sum_{j=M+1}^{K} x_{ij} \left(\frac{\nu_{ij}}{c_i} - 1 \right)$$

Expected return of service portfolio x_i of component provider i:

$$\bar{r}_i(x_i) = \sum_{j=1}^{M} \mu_{ij} x_{ij}$$

This is the performance measure of the service portfolio. Here μ_{ij} is the expected return associated with provision of component i to service j:

$$\mu_{ij} = E(r_{ij}) \text{ where } r_{ij} = \frac{\nu_j \gamma_{ij}}{c_i \lambda_{ij}} - 1 \text{ for } j = 1, ..., M$$

$$\text{and } r_{ij} = \frac{\nu_{ij}}{c_i} - 1 \text{ for } j = M+1, ..., K.$$

and r_{ij} is the random return associated with provision of component i to service j. Its randomness is connected with the uncertainties of revenues, costs and even service composition. It brings risk that the realized return will differ from the expected one. This risk should be measured and controlled.

Risk associated with service portfolio x_i of component provider i:

$$R(x_i) = \mathrm{StDev}(r_i(x_i)) = \mathrm{StDev}\left(\sum_{j=1}^{K} r_{ij}(x_{ij}) \right)$$

We take here the traditional way of financial theory to measure risk with standard deviation of portfolio return [32]. It is also possible to include modern risk measures like Value at Risk or Cash Flow at Risk [1] into the analysis. After having defined the notions of performance and risk we can now follow the approach of portfolio theory [32] in order to obtain the composition of the component provider's service portfolio. This theory looks at the portfolio selection as a trade-off between risk and performance and proceeds as follows.

1. *Construction of the efficient frontier.* Some target average return η is fixed. The risk of the service portfolio is minimized with subject to this target return, i.e. the following problem needs to be solved:

$$\min_x R(x_i)$$

$$\bar{r}_i(x_i) = \eta$$

$$\sum_{j=1}^{K} x_{ij} = 1, \; \sum_{j=1}^{K} x_{ij} \geq 1$$

The solution of this problem for all admissible values of the target return η will provide the set of service portfolios which are the reasonable candidates to be selected by the component provider i. They constitute the efficient frontier of the set of all possible service portfolios. This concept is illustrated in Figure 7.

Each service portfolio x can be characterized by the risk-return pair defined above. Therefore it can be represented as a point in the risk-return space depicted in Figure 7. The set of such points for all possible portfolios describes all existing relations between risk and return and is called the feasible set. However, an actor will seek the highest possible return among equally risky alternatives or she will seek the lowest possible risk among equally profitable alternatives. Considering Figure 7 it becomes

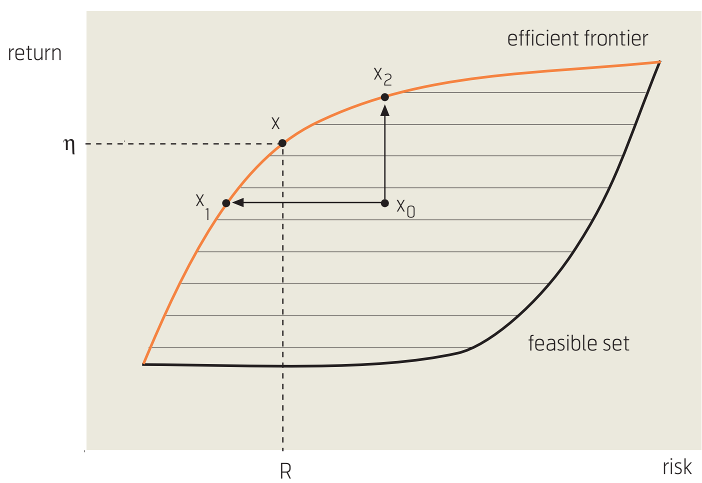

Figure 7 Selection of an efficient service portfolio

clear that some of the service portfolios should be preferred to others. For example, let us consider the feasible portfolio x_0. It is clear that portfolio x_2 should be preferred to x_0 by an agent who makes his decision on the basis of return and risk. This is because portfolio x_2 has the same risk as portfolio x_0 and a larger return. Similarly, portfolio x_1 should be preferred to x_0 as well because it provides the same return with a lower risk. Thus, portfolio x_0 is dominated by both portfolios x_1 and x_2 and should not be taken into consideration. The actor whose decisions are guided by risk and return should only consider non-dominated portfolios which constitute the efficient frontier which is depicted by the orange curve in Figure 7.

2. *Selection of the target service portfolio*. The previous step resulted in the selection of a much smaller set of efficient service portfolios from the set of all possible service portfolios. An actor selects his target service portfolio from this efficient set by choosing the trade-off between risk and return. One way to achieve this trade-off is to consider the largest risk an actor is willing to take. Suppose that the value of such risk is R (see Figure 7). Then the actor should choose the portfolio x on the efficient frontier with this value of risk. Suppose that this service portfolio yields a return η. No other portfolio yields a better return without increasing the risk. If an actor is not satisfied with the return η then she should increase her risk tolerance or look for opportunities to participate in the service provision not yet described in this model.

Service or Platform Provision Layer

Here the service or platform provider decides about revenue sharing, pricing, and the composition of the bundle of platform services. Different component providers select their service portfolios observing these decisions as exogenous inputs and having their targets described in terms of return on investment and risk tolerance. However, a service or a platform can become a reality only if the participation in its provision will be consistent with these individual targets. This means that all actors which cover the roles indispensable for provision of a particular service should have this service in their efficient service portfolio. In other words, the service portfolios of the relevant actors should be coordinated and compatible.

Thus, the service or platform provider should make his decisions in such a way as to assure this coordination and compatibility. He does this by choosing his own trade-off between return and risk similar to how it is done on the component layer. The resulting decision structure is similar to what is described in Section 3.1 and is obtained by solving the stochastic optimization problem with bilevel structure.

Architecture of the Decision Support System

We now develop a prototype of a decision support system for the assistance of strategic decisions and the evaluation of business models in multi-agent environment under uncertainty typical in telecommunications. It combines a customized implementation and model development with the use of general purpose mathematical modeling systems and commercial software. The architecture of this system is shown in Figure 8.

The system is composed of four components: data and user interface, a library of service models, a library of mathematical models and a library of solvers.

The *Data and user interface* is implemented in Excel due to its familiarity to potential users. Its purpose is to provide an easy tool for storing and changing the data that describe the service and customer properties, for the presentation of results of business modeling and for providing the capability to the system user to ask what-if questions pertaining to different scenarios. For example, the efficient frontier in Figure 7 is presented to the user through this component.

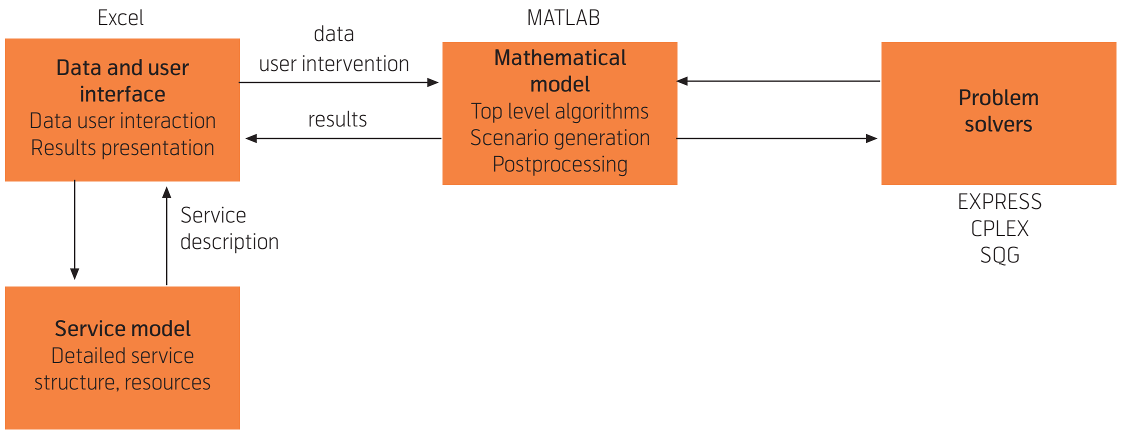

Figure 8 Architecture of decision support system for evaluation of business models of service provision

Service models provide the capability to perform modeling of advanced data services with the aim to obtain the aggregated description of the services' composition λ_j.

The *Library of mathematical models* implements the quantitative description of the business decision process of the collaborative service provision from the previous sections. It imports data from the data interface and implements the top level structures and algorithms necessary for the representation and solution of the models described above. The custom algorithms for an analysis and solution of these models are implemented in MATLAB. This component is also responsible for calling external commercial software for solving sub-problems with standard approaches.

The *Library of solvers* contains solvers for linear and nonlinear programming problems and some specialized solvers for stochastic programming problems like SQG in Gaivoronski [20].

The system depicted in Figure 4 is now in an advanced stage of development. In particular the service model component and some mathematical models of service provisioning were implemented in MATLAB.

4 Conclusions

Stochastic optimization coupled with the notions of modern investment science and game theory constitute a powerful tool for evaluation of business models and support for strategic decisions under risk and uncertainty in the multi-agent networked telecommunications environment.

Many relevant issues remain beyond the scope of this paper and will be treated in our future research. These include different actor constellations, combinations of roles by an actor, evaluation of the whole service provision platform, modeling of flexibilities and uncertainties inherent in the service provision, the life cycle of a service, and others.

Another important objective to pursue on the methodological level is to integrate approaches and findings of computational multi-agent economics and statistical mechanics of networks. Particularly relevant is the description of market trends and behavior by means of these approaches and to consider them appropriately when evaluating decisions and strategies of telecommunication companies.

5 Acknowledgement

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